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PHYSICS

COMMENTS ON THE APPLICATION OF PHYSICS TO METALLURGY

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METALLURGY as an art dates from the end of the stone age, and at that time, was a great factor in directing the course of human civilization. Metallurgy as a science developed with the modern industrial era. It has always been concerned with how man lives, rather than how he thinks, and has seldom strayed far from its purpose of improving the products and methods of industry.

It has been estimated that the United States spends 10 million dollars and has 3,000 workers in pure science, and 200 million dollars with 30,000 workers in applied science. Of the ten leading industries from point of view of manufacturing, eight are metal-working and metal-using industries. According to the Department of Commerce, the metal-working industries spend on research, on the basis of capital invested, 2.1 percent per year, being second only to the chemical industries. This shows in a rough way the magnitude of the metal-working industry, and its support of research.

It is the purpose of this article to outline the relation of physics to metallurgy, the science underlying the metal-working industry, and to point out the need of men properly trained in physics to advance the science of metallurgy.

In comparing one branch of science with another, it is convenient to outline certain artificial boundaries for each, but there is no intention to separate the two sciences, to say here physics ends, and here metallurgy begins, but rather to show those places where the two must advance side by side.

Within the last two decades, the connection between physics and metallurgy has been vastly strengthened. The metallurgist has increasingly used the methods and apparatus of the physicist, and much to his advantage. The inclusive term "metallurgy," as used today, is more likely to connote physical metallurgy,—the science which relates to the production of materials of engineering utility from the metals of commerce by alloying, by hot or cold working, or by heat-treatment,—than it is to connote chemical metallurgy, —the science which relates to the extraction of metals from their ores.

Physics and chemistry together, and more specifically the particular combination called physical chemistry, are nevertheless the sciences underlying

chemical metallurgy. Scientific chemical metallurgy is by far the older of the two branches of metallurgy, and most of those who practice it have been trained as chemists, specializing in physical chemistry. As these men were confronted with problems in physical metallurgy, they had, in general, to dig out for themselves the physical principles involved, often being handicapped by inadequate preparation in physics. In this, they found their physical chemistry to be that part of their college training most nearly related to the principles they had to deal with in physical metallurgy. This reacted upon the college courses in physical chemistry so that those courses became a better preparation for general metallurgy than those in inorganic chemistry or in physics.

This was probably not due to any intentional orientation of the physical chemistry courses, but rather to the fact that the literature of physical metallurgy, being largely written by physical chemists, came to contain much that is of value in teaching the principles of physical chemistry.

No such reaction took place upon the average curriculum in physics, which has, in general, not been oriented to take account of the existence and the importance of physical metallurgy.

Hence, the anomalous situation has arisen wherein physical metallugry is more directly a child of chemistry than of physics, and, taking it by and large, the average well-trained physical chemist is better raw material from which to make a metallurgist than is the average well-trained physicist. This is not so much due to the difference in the curriculum as it is to the point of view that, without formal precept, but rather through the personality and example of his professors, has permeated the mind of the average man who takes advanced work in chemistry compared with the one who takes it in physics.

The chemist realizes that chemistry is a vital force in the engineering advances that improve our standard of living. To him, chemistry is not an abstract science to be followed solely for sheer intellectual joy and culture; it is as well a science in the service of mankind. Even if his own attitude of mind is such that he prefers to strike out in those fields of pure research that have no likelihood of directly useful application, at least in his own lifetime, he does not look down upon those other chemists who seek to surmount the next little hill on the road of progress instead of going through the unblazed forest toward the peak of some unclimbed, and perhaps unclimbable, mountain.

The physicist is far more likely to look upon physics as an end in itself, a means for pure intellectual gratification—something analogous to the study of old Norse—, more likely to be pedantic, and to adopt a supercilious attitude toward those who are interested in practical, engineering uses of physics. Those who take this attitude usually attempt to justify it by stressing the importance of "fundamentals."

Foundations are necessary, but sky-scrapers are not built by laying a foundation, and never even putting on the first story.

To those of us who see the dependence of the metallurgy of the future upon physics, and deplore the indifference of the average professor of physics toward the lines of physical research in which metallurgy needs more informa-

tion, and toward the training of men properly equipped to carry on such research, it is encouraging to find that there is, after all, enough interest in the application of physics to justify the establishment of a new journal in which applied physics will receive the attention it deserves. It is therefore a pleasant task to point out the service that the older physics has already rendered to metallurgy, and the increased service it should render in the future. It is a less pleasant duty, but nevertheless a duty, to comment on the trend of the newer physics from the point of view of the physical metallurgist.

Metallurgy is essentially a practical science. The metallurgist is more interested, it is true, than the engineer in "why" but he is largely interested in "why" as a means of understanding and predicting "what" will take place in the preparation and use of metallic materials. Metallurgy most earnestly desires of physics an increased development of that portion of the science which deals with practical applications, and of the physicist whose aid it solicits, an understanding of and sympathy with the limitations placed upon his flights of fancy by the requirement that the result must be useful. That chemists are more commonly associated with metallurgy than physicists, is probably not due to a greater common heritage of fundamentals, for here physics undoubtedly leads, but to the closer relation of chemistry to applied science.

Before enumerating instances of the application of physics to metallurgy, let us for a moment look at some portions of physics which, were any arbitrary division of subject matter made, would possibly be assigned to metallurgy. The selection rests partly on a division of subject-matter, and partly on possession by right of conquest or continued occupation.

Metallurgy would, along with engineering, claim the field of "elasticity." The study of the action of stresses, as dependent in particular on the metals stressed, may be considered pure metallurgy. Strength, hardness, ductility and endurance under repeated stress, the routine "physical testing" of the metallurgist, are seldom adequately defined or measured in physics.

Present methods of measuring high temperatures were developed largely to supply the demands of metallurgy. Temperature is recognized as metallurgy's most important independent variable, more important, probably, than time.

Pyrometry and the calibration of pyrometers with a care only applied by the physicist to his most fundamental work is an every-day job for the research metallurgist, and most physicists would be surprised at the care taken in these matters in up-to-date metallurgical heat-treating plants.

Metallurgy looks to "modern" physics for the theoretical solution of many problems, among them the following: the strength of single crystals; the strength of polycrystalline material; the effect of distortion and slip on strength; the electrical and thermal conductivity of metals and alloys as single crystals, polycrystalline material, and cold-worked materials; the variation of all properties with temperature; magnetism; and for laws of crystal structure (perhaps dependent upon atomic structure) that may make it possible to predict the properties of alloys.

Rules, more or less empirical, would be acceptable to the metallurgist; reasons and laws in usable forms are to be preferred.

Spectrographic analysis, both qualitative, and the more recent quantitative work, are of immense value to metallurgy. Even the difficulties presented by the analysis of alloys of iron are being gradually overcome.

The metallurgist has shown a great interest in the powerful and precise instruments and methods of physics. In this connection, it might be remarked that the physicist has almost ceased making careful measurements of "physical constants" of material except as the result has important bearing on theory, or to test a new method. Where the desired end is precise data, the metallurgist has found it necessary to secure it himself. Careful measurements of thermal and electrical conductivity are now being made by many metallurgical laboratories.

We have already intimated that physical metallurgy is preeminently a field for the applied physicist if he will enter that field. But physics serves the geologist, the metal mining engineer and the chemical metallurgist as well, in finding, concentrating and smelting and refining the ores into the metals of commerce which are the raw materials of the physical metallurgist.

Geophysical prospecting utilizes a score of different devices and methods in finding ores, and these methods and devices are straight applied physics. In precision of methods and perfection of instruments, geophysics, admittedly an applied science, has far outstripped the branches of pure physics on which it is based. When useful, practical, money-making results are obtained by a scientific method, money is gladly made available for the perfection of that method. Pure science never loses by being put into practice. It is perhaps worthy of comment in passing that when it is applied it tends to be called, not physics, but physics with a prefix, or "physical" science of some sort. Physics as such thus tends to be eroded away.

Passing over many applications of applied physics in mining itself, we may take up the next step between the mine and the ultimate consumer.

Before the chemical processes of smelting or electrolytic reduction of the ore can be applied, the ore has to be crushed and concentrated. It often falls to the chemical metallurgist to supervise the work, especially since he, rather than the mining engineer, has been responsible for modern developments in concentration. Crushing and grinding by various types of machinery depend on principles of physics which have usually been handled by engineers. They have made strides in the economics and technology of the processes, but chiefly by cut and try methods. Too few physicists have interested themselves in these problems, and grinding and grindability are still fertile fields. Wear on crusher jaws and on mill balls and linings is a small corner of the huge wear problem which needs the attention of the physicist in elucidating the mechanism of wear. That rubber linings for ball mills would be as useful as they are, could probably have been predicted by the physicist long before they actually came into use, had he given the matter attention.

After the ore is ground to such fineness as will release the gangue most economically, it must be subjected to concentration, Jigging, tabling and other methods of gravity separation, as well as magnetic and electrostatic methods are obviously physics. A new era in ore concentration came in with froth oil-flotation. Present costs of production of copper and zinc would be

far greater had not flotation made it economically feasible to treat ores of extremely low metallic content. Flotation depends upon adhesion and surface tension effects whose behavior was worked out by rule of thumb. The selection of flotation oils and reagents, and of flotation methods for concentration of a given ore are not yet based on sufficiently reliable theory. With rare exceptions, the physicist has been conspicuous by his absence from this important field. If the physicist were as prone to poke his nose into what is normally thought of as the business of some brand of engineer, as the chemist is, he would long ago have seen an opportunity in the whole field of ore concentration.

Once the separation of the ore is accomplished, it must be put in shape for smelting. For example, slimes must be de-watered. In settling and filtration practice, the chemical engineer has done a good job in the field of physics without much help from physicists. Sintering practice remains in the hands of engineers, and the economics of sintering are the stumbling block on many an ore that requires fine grinding for its concentration. Agglomeration of fine ores into suitable form to charge into a smelting furnace is still too much of a cut and try process.

Coming now to the reduction of the ore itself by smelting or by leaching and electro-refining, we get more into the fields of high-temperature chemistry, and of electrochemistry, than into physics. The physical chemist holds sway here, as equilibria, rates of reaction, thermodynamics and free energy data come into consideration, and these he has adopted as chemistry though they can as readily be classed as physics.

The refinement of methods for determination of the basic data is a task best solved by those well acquainted with precise physical laboratory technique. Difficulties also enter through the inability of mathematics to get anything more out of the problem than is put in by the original postulates, and, to bring mathematical complexities down so they can be handled, the postulates often involve simplifying assumptions that leave out too many variables for the result to have a true physical meaning. Rarely can a free energy calculation be made with sufficient accuracy to be of great service as yet, because of the lack of reliable data. We do not especially suggest the field of free energy for added attack by physicists; there are as many physical chemists studying it now as its metallurgical importance seems to deserve at present.

Electrostatic precipitation of fumes produced in smelting operations not only avoids a nuisance to the neighborhood, but also often recovers material of sufficient value to show a handsome profit. Though a direct application of physical principles and methods, electrostatic precipitation has been developed largely by physical chemists.

The several metallurgical operations involving heating present problems in heat transfer which have been solved only in part. The difficulty lies not only in the mathematics involved, but in the lack of adequate theories, and usable data.

At moderate temperatures, transfer by pure conduction can be computed with sufficient accuracy from available conductivities. Conductivities at high temperatures, particularly for refractory materials, are somewhat uncertain.

Practical problems will always require solutions of Laplace's equation with special boundary conditions.

Heat transfer by radiation is extremely important in high temperature heating, such as open-hearth melting and other refining operations. The radiation constants involved are poorly known. For flames and gases, the radiation constants are scarcely known, the temperatures almost impossible to measure, and "size" and "shape" factors are difficult to evaluate.

Transfer by convection, both natural and forced, is impossible to calculate with reasonable accuracy except in a small number of cases. Results may be uncertain by a factor of ten or more. Even at moderate temperatures with simple geometrical shapes, better information on natural convection is to be desired.

A field that deserves continued attention is the physical chemistry of steel making, the campaign for facts and methods that will yield cleaner and more reliable steel. Here we meet such problems as the application of Stokes' law of settling, the determination of the viscosity of slags and many others in which the combined efforts of physicists and chemists are required.

Once the metals are produced and purified, by the methods of chemical metallurgy, and we start to alloy and form them into materials of engineering, we enter the seas sailed by the physical metallurgist.

His charts are the equilibrium diagrams given him by the phase rule, and his compass the metallurgical microscope. The determination of the diagrams is based wholly on the methods and apparatus of physics. Thermal analysis, electrical conductivity, thermal e.m.f., thermal expansion by precise dilatometric methods, and, to a certain extent, magnetic methods, are applied in working out the diagrams. The two chief tools are however, the metallographic microscope, and, more recently, the x-ray spectrograph.

Metallurgy entered a new era when the microscope became available, enabling the metallurgist to see, understand and control the changes in microstructure that produced phenomena that were previously so baffling and mysterious. This tool, supplied by physics, has proven indispensable.

The knowledge of the structure of metals, made possible by the microscope, has given us modern heat-treatment methods without which modern aircraft and automobiles, to say nothing of most other methods of transportation as they are today, would be impossible. Optics has been a worthy handmaiden of metallurgy. Not so long ago, 500 diameters was high magnification. Today, 1500 diameters is common in any decently equipped metallurgical laboratory, 5000 not uncommon and 15000 to 25000 have been reached. While extreme magnification is seldom a game worth the candle, there are some cases where it is, and better resolution at high magnifications is being sought by use of ultraviolet light.

Hand in hand with the better understanding of the principles of heat-treatment gained by means of the microscope has come the necessity for precise pyrometric control in order to obtain the best, and sufficiently consistent, results from heat-treatment. Thermoelectric pyrometers, actuating automatic devices for the control of electrical resistors or of burners, have taken the process of heat-treatment away from the "man at the fire," and made it a

mechanical process. Physicists and firms that formerly specialized in laboratory apparatus for physicists, are largely responsible for this.

Our best understanding of the important processes involved in the hardening of steel by quenching is due to studies with the electro-cardiograph, which can follow and record the changes taking place during cooling at as rapid a rate as 10000° C per second.

Mechanical testing is next door to physics, though usually considered in the field of the mechanical engineer. Demands for better metallurgical information are, however, throwing it back into the field of the physicist. Metals fail in severe service due to corrosion, to "fatigue" under repeated stress, to abrasion and to deformation under stress at high temperatures. Corrosion and its prevention may perhaps be left to the chemist, though the electrochemical theories of corrosion that are throwing most light on the problem are as much physics as chemistry. Understanding of the mechanism of failure under repeated stress above the endurance limit, of strengthing by understressing below it, of the behavior of metals in resistance to impact, and the propagation or lack of propagation of a notch, involve problems of energy absorption and hysteresis that the physicist is best qualified to deal with.

"Creep" under high temperature loading, brings in concepts of the phenomena of plastic flow in metals for whose clarification we must certainly look to men trained in physics.

Progress in the power plant industry, in oil refining, in some chemical industries, in many parts of the metallurgical industry itself, in the automotive industry, and various others, is quite definitely limited by restrictions due to the inability of present alloys to withstand without permanent distortion the loads the engineer wishes them to stand at high temperatures. Hundreds of thousands of dollars have been invested in "creep test" equipment in which metal specimens are held under load at closely controlled temperatures for long periods, not infrequently for several months, and the stretch measured to a few millionths of an inch in order to yield data for design of turbines, cracking stills, apparatus for ammonia synthesis, etc.

It requires training in precise methods of physical measurement to design, construct and operate the precision instruments required in present-day physical metallurgy. Still more is demanded good training in physical principles to correlate and systematize the facts and develop the underlying theories that will make these facts capable of being understood and memorized. Few physicists are giving attention to the metallurgical problems in which their help is most needed. Take for example, the pressing problem of the wear of metals, and its cognate problem of lubrication. Abrasion, whether by nonmetallic materials or from metal-to-metal contact, is an important, but little studied, mode of failure of metallic parts. As corrosion becomes more adequately combated by corrosion-resisting alloys, and by more permanent protective coatings, so that things do not rust or corrode away so rapidly, more attention will be paid to keeping them from wearing out.

The mechanism of wear is but poorly understood, and methods for the determination of wear resistance are in their infancy. No one knows just what properties are required in a bearing metal, nor why the alloys used for bear-

ing metals are suitable for that purpose. The more the matter is studied, the less sound appear the principles so glibly stated in the ordinary text book that gives a paragraph or so to the subject. Some engineers think they have some idea why lubricants are lubricants, but when one goes from the lubrication of bearings to that of wire-drawing dies, and of dies for deep-stamping of metals, he finds that the principles of lubrication are not yet sufficiently established to allow much extrapolation. There seems to exist a possibility that combinations of alloys not now considered as bearing metals and materials not now considered as lubricants might prove as good or better than the present combinations. In this field we are almost without quantitative methods of test of the real variables involved, and the point of view of the physicist seems the one best suited to produce what is called for.

In the intentional removal of metal by machining and grinding, so important in the shaping of metals for industrial use, we again lack a yardstick. Metals are "machinable," "difficulty machinable," or "unmachinable," but the property is not yet expressible in satisfactory quantitative terms. Mechanical engineers and some few testing engineers are struggling with the problem, but here again is a need for competent physicists to bring to bear their point of view.

The shaping of metal by hot or cold rolling, forging, wire-drawing, swaging, spinning, pressing, stamping, and so on, brings in, in accentuated fashion, the problems of plastic flow of metals. Engineers who have added to our information of the fundamentals involved have relied largely on the methods of physics.

The time-honored method of forming metals by casting has many problems of gating that are pure hydrodynamics. The molding and core sands used in the foundry have in recent years been given much attention, with great results in reduced percentages of defective castings and lowered costs through the reuse of used sand. The making up of artificial molding sands with suitable strength from sand with insufficient natural bond by the use of suitable plastic clays brings in questions of the properties of mixtures of grains of varying sizes, and of the measurement of particles of submicroscopic fineness. The permeability of the sand brings in problems of "close packing," and of how to combine suitable strength, and yet maintain the proper voids to produce the necessary permeability. Some of the properties of sand are evaluated by crude but rather effective methods of physical measurement. Interest by physicists in the fundamental problems should still further improve the situation.

Fabrication of metallic articles by welding is growing apace, and real strides are being made in development of mechanical processes of welding that get away from the danger of imperfect and unreliable hand welding. The new welding methods that utilize hydrogen are ascribable to the physicists.

The need for the evaluation of the soundness of welds, and the menace of rail failures through invisible internal transverse fissures has led to the development of various non-destructive methods of testing, utilizing electrical conductivity, magnetic properties, and the effect upon sound vibrations detectable by the stethescope. Radiography also serves to disclose hidden defects if they are of sufficient size.

An extremely useful metallurgical tool, the high frequency furnace, devised by a physicist of the old school, has greatly facilitated metallurgical laboratory investigations, and has more recently entered the field of commercial melting.

The history of the advances in physical metallurgy has been very closely connected with the development of physical devices for precision control of metallurgical operations and for precision measurement of physical properties. Development of metallurgical theories to explain old facts and predict new ones has spurted each time a new tool was available. The perfection of methods of preparing single crystals opened up studies that throw light on the engineering properties of polycrystalline materials.

Among the newer tools that physics has given to metallurgy is the x-ray diffraction apparatus, which, as previously stated, has helped a great deal in more exactly establishing the equilibrium diagrams of alloys. It has distinct limitations because some of the crystals that are of vast importance metallurgically, cementite for example, are so built up that they do not give good x-ray reflections, and their presence may readily be missed. It often clears up moot points, and does so with emphasis when it gives positive information. When it gives negative information, this may be true negative information, or may be merely the insensitivity of the method.

But on the whole, the picture it gives of the arrangement of atoms in crystals, the information it produces as to lattice expansion and distortion, the light it throws on preferred orientation in cold working, on crystal size, and the things it suggests even if it does not prove the point, are most helpful, and it is becoming a useful adjunct to the metallurgical microscope.

Many physicists and some metallurgists might be inclined to rate the development of x-ray methods of attack as even more important than it is here rated. But its availability has not had exclusively wholesome effects.

This and spectroscopic studies on the "fine structure" of matter came along, at about the same time that "modern" physics became largely concerned with quantum theories, relativity, and the Heisenberg uncertainty principle. Physicists flocked into speculation, and into such meager experimentation as is possible, upon atomic and sub-atomic phenomena, almost to the exclusion of thought and experiment on the properties of matter in bulk. Since the x-ray peers into the minute, it has become a very popular tool of the modern physicist. Since metals are conveniently studied by the methods of x-ray crystallography, the metallurgist has profited by the work of the physicist in this direction. He has not, however, profited much by the mathematical speculation on atomic and subatomic structure that has followed in its train.

We should not be understood to be objecting to having some physicists constanting searching for those underlying laws of atomic and sub-atomic structure whose full understanding might make it possible to calculate by mathematics alone, and without experimentation, all possible physical properties of materials. Remote as this prize may be, a few physicists, who have a suitable mental armament might well continue to attack the citadel of the "new" physics.

But, from the point of view of the metallurgist, we deplore the over-emphasis being put upon the new physics. It appears that graduate instruction in physics is concentrating far too much on mathematical speculation that approaches metaphysics, and the old discussions as to how many angels can dance on the point of a pin.

Applicants for positions in physical metallurgy proudly cite the number of courses they have taken in atomic physics, and hand in reprints of publications along x-ray lines. But when one quizes them on their understanding of the physical phenomena of matter that affect its engineering usefulness and their familiarily with precision physical apparatus in general, one finds a woeful lack of understanding of the older physics. What metallurgy needs is not the present crop of lop-sided atomic specialists, but general practitioners of physics who are interested in, and keep somewhat abreast of the newer physics, but at the same time do not consider the law of gravitation beneath their dignity.

That thermionic phenomena connected with metals are the basis for radio and all vacuum tube engineering, and are sub-atomic matters, is true. How much "modern" physics has entered into advancement of theory and application we will leave to those better able to discuss it. But we may at least point out that the impetus toward such rapid progress in these lines has come directly from those concerned with making money by the application of the principles.

Those professors and deans of physics who steer their graduate students away from the older engineering physics and follow exclusively the newer atomic physics like a flock of sheep, because it is the fashion set by those in other universities, might well ponder whether they are not doing a dis-service to their students and their science. If automobile makers had put all their energies on the improvement of tires, and had left all other parts of the car as they were ten years ago, they would have done very much what most physics departments have been doing. A better sense of proportion would steer them away from the present neglect of many branches of physics which need renewed study with modern tools.

The general who masses all his troops on one salient, and neglects the rest of the battle front, must succeed in breaking through on that salient if history is to praise him. A steady advance all along the front allows those behind the lines to advance too. Without such a general advance of physics, metallurgy must itself show little advance. It begins to question the soundness of the strategy of the physics general staff.

Metallurgy wants applied physics. It wants more of the fundamentals that can be applied later, and it is glad to see atomic physics included in those fundamentals; but it emphatically does not want to see an undue proportion of each succeeding crop of Ph. D. physicists emerge with less than a smattering of that type of physics which is demanded for the advancement of metallurgy. When everybody is leaving the hardware business to go into the perfumery business, is a promising time to take up the hardware business.

Those outside of physics may perhaps appraise the situation better than those who are inside. Here are some recent comments on atomic physics.

Kuntze[1] says, "The direct application of the laws of atomic physics to the strength of crystalline materials has so far been unsuccessful. Hence a technical examination of the strength problem, such as is being carried out in the laboratories of governments and of private industries, is necessary."

Andrew[2] comments, "Any ideas based on atomic theories which are in themselves evanescent, cannot but be likewise".

A non-technical observer[3] may next be cited. He states that Henry Adams in 1909 predicted "a break-down of human thought" about 1921, and says "In 1909 to have predicted that we should reach the end of human reason seemed absurd—but it has come about, and the physicists have themselves been most unwillingly forced to admit that we have reached a point beyond which our reason is now powerless to proceed in our investigation of the structure of the Universe, a blank wall which marks the end of knowledge, and on the other side of which the categories of human reason no longer hold sway".

Knox[4] freely applies the term "meta-physics" to modern physics. He says, "The day of the engineer-scientist is done; it ended with the close of the century that is past, and the higher flights of fancy into the realm of meta-physics are not for him as engineer. So the engineer with a sigh, must turn back to the world of Euclid and Newton, leaving the multi-dimensioned geometries, the quantum mechanics and the rest, to the pundits. To them he must leave the exposure of that part of reality that is comprised in the field of physics."

Lest this be deemed too severe, let us here insert a comment from a leader in physics itself. Dushman[5] says "A theory must enable us to test it by experiment, or else it is theology".

Where to get our future "general practitioners" of physics, the "engineer-physicist" of Civil Service parlance, is a problem that perplexes thinking metallurgists. Most of our engineer-physicists are middle-aged. A young physicist of this stamp is a rarity. One might paraphrase the attitude of most faculties of physics to whom the question might be put, in some such fashion as this.

"Well, what of it? We recognize no responsibility to train men in theoretical physics with the idea that such training is a foundation for the practical application of physics to industry. Theoretical physics is a system of logic, engaged in by those whose mental make-up is such that they enjoy it, and to whom it is quite immaterial whether anyone ever builds any practical results upon the theoretical foundation. That is, the study of atomic physics is, to us dyed-in-the-wool theoretical physicists, much like Roy Chapman An-

[1] W. Kuntze, Die Bruchgefahr bei metallischen Werkstoffe. Mitt. a. d. deutschen Materialprüfungsanstalten. Sonderheft 14, 1930, pages 60–61.

[2] J. H. Andrew, Effect of Certain Elements on Segregation of Cementite and Its Effect on the Modification Process. Metallurgical Club Journal, Royal Technical College, Glasgow, 1929–30, No. 7, pages 16–17.

[3] J. T. Adams, The Adams Family. Little, Brown & Company, 1930, p. 345.

[4] H. H. Knox, Correspondence on Einstein's special theory. Mining and Metallurgy **12,** April, 1931, page 206.

[5] S. Dushman, quoted, News Edition, Industrial and Chemical Engineering **9,** April 19, 1931, p. 98.

drews' search for fossil dinosaur eggs. The minute a physicist begins to 'talk sense,' and make practical application of his physics, he becomes, for lack of a better word, an engineer. If there is any responsibility for the training of engineering physicists, it is not ours, but rests on the engineering faculties."

One English university has a more helpful attitude. Southwell[6] says, "In recent years, the physicists have concentrated more and more the whole strength of their army upon one stupendous effort, an attack on that citadel wherein lies hid the ultimate mystery of atomic structure. To this adventure almost all their best students now are dedicated. Hydrodynamics, heat-conduction, elasticity, all the 'field physics' of the 19th century, win but a few men here and there.

"Here there is surely a field for the academic engineer. Since his concern is always with matter in bulk, and since the behavior of matter in bulk has, it appears, small interest for the physicist compared with the behavior of the individual atom, let him take this for his province and follow, if he can, the lead of Maxwell, Kelvin, Rayleigh, Osborne Reynolds, Stokes and Lamb.

"These, with Fourier, Helmholtz, Gauss,—all the galaxy that was the glory of 19th century physics,—become eligible for our engineering team."

That some university somewhere has this point of view, is encouraging. But in America, graduate students in engineering are very rare, while graduate students in physics are fairly plentiful. So if there is any hope at all, it must come from a changed attitude on the part of faculties of physics.

We have tried to show how dependent modern metallurgy is upon physics, and to touch briefly upon some of the fields of physics in which the metallurgist looks for aid from the physicist. We had begun to be pessimistic about the prospects of getting the aid in the future that has been rendered in the past.

One of the maxims carved on the facade of Battelle Memorial Institute, and to which we subscribe, is "The life of science is in its practical application." But, to have much to apply that will help metallurgy over the rough parts of the road that is in sight, physics must regain its interest in research along some of its older branches. We are not arguing for the "practical" trade school type of education; far from it, for what is needed is the broadest possible foundation and the ability to build upon it. Good grounding in the theories and methods of the older physics is a big asset in the raw material for future physical metallurgists. Metallurgy would be happy if physics would prepare men to become "metallurgical physicists" instead of making it create its own "physical metallurgists," and chiefly from the material chemistry provides. It does not ask the colleges for intense specialization along metallurgical lines, but rather for the training of applied physicists, engineer-physicists, who have a proper background for specialization in metallurgy when the metallurgical industries call them into service.

May the establishment of "Physics," and its acknowledgment of the fact that applied physics is dignified and worthy, mark the commencement of a new era, or rather, the re-establishment of an old one, an era of renewed service to metallurgy. There is no field in which physics can render greater service, if it will.

[6] R. V. Southwell, The Place of Engineering Science in University Studies. Inaugural Lecture, University of Oxford. June 17, 1930. Clarendon Press.

THE HUMAN VALUE OF PHYSICS

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IN CONSIDERING the human value of physics, we think at once of the great changes in man's mode of life that have resulted from its applications. These are of great importance. Yet perhaps of greater significance is the change that the growth of physical science has made in man's attitude toward life. Increased knowledge of physics has ushered man into a new world, with a challenge for him to shape his destiny on a more heroic scale.

Physics, and Man's Attitude toward Life

It was in the effort to find man's place in nature and to lay a reliable basis for life that the first studies of physics were undertaken. "Of what and how is the world made," was the problem set by Thales, the father of Greek science. To him and his successors science was of value if it would enable men to attain a more satisfactory philosophy of life. Likewise Pythagoras, on the basis of his broad knowledge of the physical world, established a religious order whose aim it was to bring about the rule of reason and morality. This ambitious objective is as yet far from achievement. Nevertheless, the many benefits that have come through learning the laws of nature have had their effect in influencing men to organize their lives on the basis of tested truth.

It is frequently overlooked that the scientific approach to life has not always been acceptable to thinking men. There was the historic protest of Socrates against the science of the Atomists, which he felt destroyed the basis of morality. In the hands of Plato and the Neo-Platonists this protest led to an anti-scientific philosophy which overwhelmed struggling Greek science and opened the way for the flood of oriental magic and mysticism which followed upon Alexander's conquests. Then came a thousand dark years during which credulity in all forms of magic and witchcraft prevailed. Man thought himself in a world governed by the whims of demons and angels.

We owe to physics and astronomy the rescue of Western civilization from these depths of superstition. Galileo's studies of dynamics as well as his telescope showed that facts were to be found by observing Nature ratner than by searching ancient writings. Newton's success in solving the riddle of the planets established in mens' minds the fact that we live in a world of law. Great improvements in our welfare have come through applications of the principles of physics. It has thus become evident to all that if we would make the best of life we must learn Nature's laws and use them as our guide.

It is perhaps this lesson of the need for knowledge of the truth and for the willingness to adjust our lives to this truth which is the great contribution of science to humanity. Guided by the truths of physics, the inventor and the engineer have freed us from much of the slavery of physical drudgery. Medi-

cal science, taking its lead from physics, and using the tools it has supplied, has gone far in freeing us from disease and pain. Seeing these successes of science as applied to other aspects of life, men willingly control their habits in accord with the laws of hygiene, and a healthier nation results. It gradually becomes recognized that human relations are also subject to natural laws. These form the fundamental basis of morality. Since the days of Pythagoras, it has been recognized that a world of law implies a life of self discipline if that life is to reach its highest development. Thus in an era when many old conventions become outworn, the clear teaching of science is of especial value: If man will learn the truth and abide by it, that truth shall make him free.

In these fundamental contributions of science to humanity, physics, due to the definiteness of its achievements and the unemotional character of its laws, has always taken the leading place.

A Brief Summary of the Applications of Physics

Enumerations of the specific contributions of applied physics to the needs and comforts of life are trite, but none the less significant. It is possible to include only the barest outline of some of the great inventions which have made possible our modern world. Every branch of physics has made its contribution:

In the field of mechanics, such elementary machines as the lever and the inclined plane date from prehistoric times. But their full usefulness could not be realized until Archimedes, the great physicist of Syracuse, learned the principles of their action. This led to his invention of the screw jack, various pulley combinations, and other machines, which have been of inestimable value to engineers. Also, without a knowledge of Archimedes' principle of buoyancy, the great ships which carry the commerce of a modern world could not be built.

For our knowledge of the properties of moving bodies we are indebted chiefly to Galileo and Newton. This knowledge came primarily from studies of ballistics and of the motions of the planets, whose practical value might today be considered minor. Yet this was the necessary prelude to such homely inventions as the pendulum and balance wheel clocks, which have regulated the life of men for centuries, and is essential for the design of the motors and the fast moving vehicles of an age of rapid transportation. It should also be noted that without these laws of motion as a starting point, there could have been no hope of understanding the discharge of electricity through gases, which has played so large a part in recent applications of physics.

In the realm of heat, the steam engine perhaps comes first to mind. Its invention by Watt is frequently spoken of as a mere mechanical improvement over Newcomen's impractical device. In truth, his great advance was the condensing chamber. This resulted from his thoroughly scientific study, made in a university laboratory, of the variation of steam pressure with temperature. His work was carried further by Carnot, Joule, Helmholtz and Kelvin, whose thermodynamic formulas have made possible the highly efficient engines and turbines of today. But such studies lead also in other directions. By operating

an engine backwards, it becomes obvious from these formulas that heat may be removed from a body, and refrigerating machines are a result. Thus is inaugurated a new era of food storage and preservation.

The physical basis of musical sounds was studied by Pythagoras at the very beginning of science. It was not until the exhaustive work of Helmholtz, however, that it was thoroughly appreciated how every characteristic of a tone has its counterpart in the associated air wave. It was this fundamental knowledge that paved the way for Edison to invent his phonograph and Bell his telephone.

The optics of lenses on the basis of which Galileo made his telescope was developed by the great Arabian physicist Alhazen. It was necessary however for Young and Fresnel to show the wave character of light before the powerful microscopes of the present day could be constructed. It is doubtful whether any one thing has contributed more to the development of the modern science of medicine than has this perfection of the compound microscope. Thus to the establishment of a fundamental physical theory, a healthy humanity is largely indebted.

In the physicists exploration of the spectrum he is not however satisfied with a knowledge of visible light. He presses beyond and studies the ultraviolet, which not only helps him to learn the secrets of the atom, but also gives him new means of keeping his health during sunless winter days. Exploring still farther he finds the x-ray, and man is supplied with an important means of surgical diagnosis. An engineer, however well trained in accepted technique, would have studied available principles in vain for a means of revealing the bones of a living man. It required the knowledge of new facts, such as the physicist is trained to seek, before the surgeon could see to set a broken arm.

One has merely to mention the word "electricity" to bring to mind innumerable services of this genie of the wires. For centuries magnets and Leyden jars have been familiar to the physicist. Yet within the life-time of men now living Faraday and Henry laid the scientific basis for the modern electric generator and motor. This had to be combined with the wave-theory of light before Herz and Marconi could produce a wireless message, and the electron must be found before the radio or long distance telephony could be a success. The genie of the wires has now become the genie of the lamp; an electric spark makes possible the gas engine and the automobile which gives many persons a life of wider interest; a nation's heart beats as one because electric nerves keep its people in constant touch with each other. Now we are worried lest the electric power companies will control our national development—which only proves how vital to our national welfare has become the wide distribution of electric power.

Thus every field of physics has contributed its great share to the welfare of mankind.

The Human Significance of the Applications of Physics

Perhaps, however, the effect of these contributions on our mode of life would become more evident if we should consider in greater detail the origin

and influence of specific inventions. A single example must suffice. Let us take the familiar one of pasteurized milk. Pasteur was led to this invention by his study of the chemical changes produced by microscopic organisms known as enzymes. These reactions were followed by means of the peculiar optical property of rotation of the plane of polarization of a light ray. Thus both microscopes and polariscopes were needed, the development of which involved a detailed knowledge of the laws of light. But still nothing could have been done had not Galileo and Fahrenheit in previous years invented thermometers for their physical studies; for accurate temperature control was essential. As a result of this invention wholesome milk can now be shipped from great distances and can be distributed to millions of children. It means good food at low cost in large centers of population.

The cumulative effect of thousands of inventions of this kind is enormous in altering our mode of living. Marco Polo's account of China in the thirteenth century shows clearly how far Europe was behind the Orient of his day. Yet in seven short centuries the leadership has passed to the western world. Certainly the most potent factor in this great change has been the rapid development of western industry under the stimulus of a series of inventions made possible by a growing physical science.

Will the Developments of Physics Continue?

If further industrial developments are to continue at a rate comparable with that of the last century, it will be necessary for additional fundamental facts of nature to be found. It is well recognized that the discoveries of yesterday form the basis for the inventions of today. It is likewise true that today's inventions will not reach their greatest usefulness unless we spend tomorrow in their scientific study and development. Consider for example the electric motor, the telephone, or the radio receiver—each invention based upon a long line of scientific discoveries which had become a background of information, but also each one followed by a long and detailed scientific study before its usefulness could be fully realized. It is because of their recognition of the importance of the latter stage of scientific development that the great industries have introduced research laboratories for the study and development of their products. It is however only those industries with the widest interests which can support the fundamental searches for the new facts on which new inventions will be based. By far the greater part of such work is being carried on by the universities, and must be supported by the public if it is to continue. This is only right, since in the long run it is the public which is rewarded by the fruits of such research.

Is it not possible, however, that physics has practically reached the limit of its development, and that by the economic law of diminishing returns research in this direction will become less and less fruitful? There was a time toward the close of the last century when physicists themselves commonly expressed this thought. Then came in rapid succession the discoveries of x-rays, radioactivity, and the electron, and physics research became the most fruitful branch of science. With the more recent developments in relativity

and quantum theory, it is probably fair to say that it has held this leading place even in competition with the rapidly developing biological sciences.

Now that electrons, protons and photons are discovered, however, and the relativity and quantum theories are to a large extent developed, what shall we expect of the future of physics? Of course there are details of these theories to be worked out and more precise information about the three fundamental elements to be gathered; but this is obviously along the line of diminishing returns. In the year 1900 Lord Kelvin saw two small clouds in the physics sky, each no larger than a man's hand. One was the Michelson-Morley experiment, and the other the radiation from hot bodies. Rapidly these clouds grew into thunderheads as the theory of relativity and the quantum theory developed. Now that these storms have passed, the physics sky is found to be clearer than ever. Nevertheless two new clouds are beginning to form. On one horizon appear the cosmic rays, bringing messages perhaps from interstellar space. What are they, whence do they come, what message do they bring? On the other horizon is the nucleus of the atom, known to contain fabulous amounts of energy which can under suitable conditions be partly released. What is it, and how may it be controlled? Newtonian mechanics does not describe its actions, nor apparently does the quantum theory. The time is ripe for a new and fundamental discovery in this field, and who can guess where it will lead?

Physical Science Vital to Civilization

Whatever may be the future of these developments, physics remains the foundation of all natural science. Its principles form the basis on which astronomy and chemistry, geology and biology are built. An adequate knowledge of cell growth, of physiological processes, of the basis of life itself, can only be secured by cooperation of physics and biology. Such cooperation is rapidly growing, and important advances in the health of mankind are resulting. In a civilization based on science it is essential that physics, the foundation of science, be kept vigorously alive.

Let us note again, however, that great as is our debt to physics for its practical benefits, of perhaps even greater human value is the spirit of loyalty to truth which has been taught by centuries of physical science. In the words of President Hoover:

"Scientific research means more than its practical results in living comfort. The future of the nation is not merely a question of the development of our industries and of reducing the cost of living, or multiplying our harvests or even of larger leisure. We must constantly strengthen the fibre of national life by the inculcation of that veracity of thought which springs alone from the search for truth. From its pursuit we shall discover the unfolding of beauty, we shall stimulate the aspiration for knowledge, we shall ever widen human understanding."

Truly, the hope of civilization lies in the advancement of the spirit of science.

CAPILLARY RISE IN SANDS OF UNIFORM SPHERICAL GRAINS

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Abstract

Uniform spheres packed in regular array form a non-cylindrical cyclic capillary, characterized by a maximum and minimum capillary rise with intermediate positions of possible equilibrium. In practice spheres may be packed to a variety of porosities P thus requiring a mixture of regular and irregular pilings arranged in a very distorted pattern. However the meniscus is also distorted to conform in a general way with the distortions of the lattice. Accordingly positions of maximum and minimum rise may be expected. The meniscus for maximum rise tends to pass through the plane of centers of neighboring spheres. Slight deviations from this condition due to the rise at sphere contacts are shown to be of minor importance. Any piling may be treated statistically as a hexagonal array with a spacing $2r+d$ where d is computed to give the observed porosity. In such a system three types of cell occur with a definite frequency, and these cell types are assumed present in the meniscus with the same frequency distribution. Hence it is possible to evaluate $pr/a=\rho ghr/\sigma$ where p=perimeter, r=grain radius, a=area of pore opening, g=acceleration of gravity, σ=surface tension, ρ=density and h=capillary rise. The final formula so derived reduces to

$$\frac{pr}{a}=\frac{\rho ghr}{\sigma}=\frac{2}{[0.9590/(1-P)^{2/3}]-1}.$$

This agreed with experiments made with several sizes of grains, porosities, and liquids. The minimum rises were also determined but a satisfactory interpretation in terms of a model has not been effected.

THE capillary rise of liquid in sands involves a consideration of two fundamental conceptions, the nature of the meniscus-grain contact and the mode of packing. Whatever packing is assumed, the regular arrangement of grains is seriously distorted so that the distribution is actually of a random nature. If a horizontal plane be passed through an extended section of sand, the pore opening is independent of the position of the plane. Hence geometrically, we have a complicated cylindrical capillary tube of uniform cross-section and perimeter. Some time ago the authors attempted to treat this problem by use of the above conception, and recently a comprehensive paper of a similar nature, has been published by Inaba.[1] Unfortunately the experimental phenomena do not follow such a simple mathematical scheme. Although a vertical column of sand geometrically is a cylindrical capillary, the meniscus surface, determining the height of rise, must be considered from the microscopic rather than from the macroscopic standpoint. The liquid surface

[1] Inaba, J. Mem. of the Col. of Eng., Kyushu Imp. Univ., Fukuoka, Japan **5**, 93–119 (1928).

in each pore, for maximum rise, tends to approach in general a position determined by a plane parallel to the centers of neighboring spheres and hence the entire meniscus presents a very complicated warped surface. The integrated effect of the elements of this surface determines the height of rise.

One of the consequences of the microscopic consideration of the problem is that many positions of equilibrium exist in an actual sand whereas the

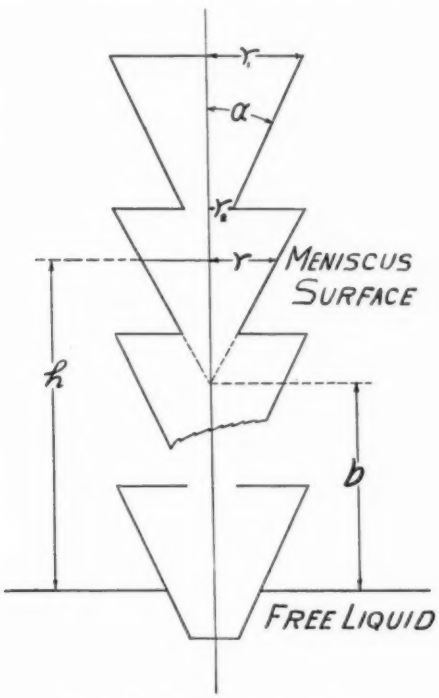

Fig. 1. Multiple conical capillary.

macroscopic picture, as described above, leads to a single height of rise. A single pore cell is a non-cylindrical capillary, and the general type of phenomena which a connected series of pores presents, may be qualitatively illustrated by the behavior of liquid in conical tubes connected as shown in Fig. 1. The dimensions are assumed such that the capillary rise involves a considerable number of cells. The equilibrium height h is given by the expression

$$h = \frac{b + (b^2 + 4A)^{1/2}}{2} \tag{1}$$

where $A = 2\sigma \cos^2 \alpha/\rho g \sin \alpha$ and σ = surface tension, ρ = density of liquid, g = acceleration of gravity. Complete wetting or zero contact angle is assumed. The quantity A is a constant while b is a parameter determining the position above the free liquid of the cell containing the meniscus. Accordingly there exists a height h corresponding, within limits, to any arbitrarily chosen distance b. The maximum h occurs when b is such that the

meniscus lies at the narrow entrance to one of the cells while the minimum h (for a rise of at least 1 cell) occurs close to the maximum cell opening. A position of equilibrium also exists in every cell between these two limits. If such a capillary be placed in a liquid the meniscus will rise only to the minimum value of h. If on the other hand the entire capillary is initially filled with liquid, the meniscus will fall to a final position determined by the maximum value of h. Intermediate positions are occupied only as a result of external agencies such as forcing the liquid up from the minimum position by pressure on the confined free liquid.

In a regular piling of spheres the pore opening, perimeter of contact and the angle at the height z between the plane tangent to a sphere and the vertical, undergo cyclic changes as z increases. Hence we have a condition analogous to, but more complicated than, that of the conical capillary discussed above, and multiple equilibrium positions are to be expected.

The condition for equilibrium of the capillary surface is

$$\int \rho g z \cos \phi dS = \int \sigma \cos \theta ds \tag{2}$$

where z is the height of an element, dS of the meniscus above the free liquid, ϕ the angle which the normal to this element makes with the vertical z-axis, ds an element in the meniscus-grain contour and θ is the angle between the vertical and that normal to ds which is tangent to the meniscus.[2] The integrals extend throughout the entire capillary surface and contour respectively, contained within the column of packed spheres.

Even in a coarse sand pore the capillary rise h is equivalent to a very large number of grain radii r. But for regular packing, over the meniscus, the quantity z can not differ from h by more than a single grain radius. Hence in Eq. (2) z is practically constant over the meniscus and can be replaced by h. The integral $\int \cos \phi dS$ is then simply the area of the projection a of the meniscus on a horizontal plane. Thus the left hand term of Eq. (2) reduces to $\rho g h a$.

For a conical or other capillary possessing circular symmetry the expression $\int \sigma \cos \theta ds$ reduces to

$$\int \sigma \cos \theta ds = \sigma p \cos \theta \tag{3}$$

where p is the perimeter of the section involved. No such symmetry occurs in the pore cell formed by three or more adjacent grains; hence the liquid-grain intersection does not lie in a plane but rises near every point of contact between spheres. Only the vertical component of the surface force is effective in sustaining a liquid column. As the liquid rises in the closer parts of the cell the value of $\cos\theta$ along the perimeter decreases while the meniscus contour or perimeter p increases. These effects are opposed in their influence, the former tending to decrease and the latter tending to increase the upward component of the capillary force.

[2] See any treatise on capillarity such as Winklemann's Handbuch.

The net effect however can be readily shown to be such that the upward force, and hence the capillary rise, is decreased by any deviation of the perimeter from the equatorial plane. Thus in Fig. 2 let ds be an element

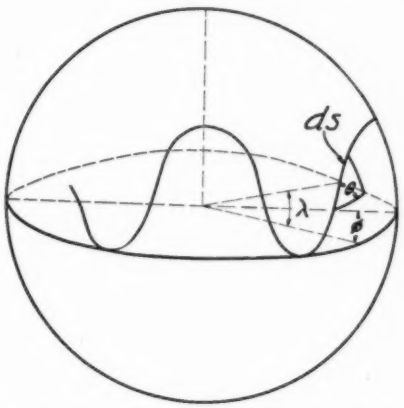

Fig. 2. Distortion of meniscus by contacts.

of the actual perimeter having the latitude λ and longitude ϕ. The inclination θ of ds to the latitude is given by

$$tg\theta = \frac{d\lambda}{\cos\lambda d\phi}. \tag{4}$$

Since the vertical force for complete wetting is $dF = \sigma ds \cos\theta \cos\lambda = \sigma r \cos^2\lambda d\phi$, the total upward force is

$$F = \sigma r \int_0^{2\pi} \cos^2\lambda d\phi \leqq 2\pi\sigma r. \tag{5}$$

The magnitude of $F - 2\pi\sigma r$ was investigated by two methods and found to be small.

First the integral in Eq. (5) was evaluated for several analytic curves resembling actually observed contours. For example the distortion of the meniscus contour by contact with six surrounding spheres in the same plane, a most extreme condition, was assumed to be represented by

$$\lambda = \lambda_0 \cos 6\phi + \lambda_0. \tag{6}$$

Substituting this in Eq. (5) one obtains

$$F = \sigma\pi r[1 + \cos 2\lambda_0 J_0(2\lambda_0)]. \tag{7}$$

For $2\lambda_0 = 9°\ 10'$, $F = 1.98\pi r\sigma$, and for $2\lambda_0 = 27°\ 30'$, $F = 1.84\pi r\sigma$. These values deviate from the maximum value of $2\pi r\sigma$ by only 1 and 8 percent respectively.

The second method for investigating the effect of distortion of the contour was experimental as follows. The force required to pull a single small

sphere from a liquid was compared with the forces required to pull various groups of spheres arranged in a plane pattern.

In Fig. 3, for a single sphere, let z be the height of an element of surface dS of the sphere above the plane of the free liquid and θ the angle between

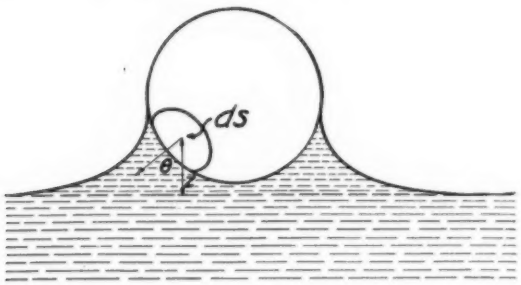

Fig. 3. Withdrawal of sphere from liquid.

the normal to the element and the vertical. The net force tending to push the sphere into the liquid, assuming the weight of the sphere as counterbalanced, is

$$B = \int \rho g z \cos\theta dS \tag{8}$$

where the integral is extended over the wetted portion of the sphere. This is a negative buoyancy arising from a lowered pressure under a concave liquid surface. For a single sphere the wetted area is confined to the lower hemisphere. Hence the force K required to pull a single sphere from the liquid surface is

$$K = 2\pi r\sigma + B. \tag{9}$$

When several spheres, in contact and in the same horizontal plane, are removed from the liquid we have in addition to the forces given by Eq. (9), a term B_c involving the correction to the buoyancy B, due to the increased area of wetting at contacts, and a term F_c arising in the distortion of the meniscus contour. Hence the total force for n spheres is

$$K(n) = n(2\pi r\sigma + B) - n_c(B_c + F_c) \tag{10}$$

where n_c is the number of contacts. The correction F_c is positive for the reason discussed above. The correction B_c is also positive as may be seen from Eq. (8) noting that $\cos\theta$ changes sign in passing across the equatorial plane. If the curve representing $K(n)$ as a function of n is a straight line passing through the origin, then $B_c + F_c = 0$ and the terms individually must vanish.

Using a DuNouy tensiometer modified by a side arm for balancing out the weight of the spheres, the forces $K(n)$ required to pull groups consisting of 1, 2, 3, 4, 6, and 9 spheres in both hexagonal and square array were measured. The curves of Fig. 4 represent the observed data and since they pass through the origin, to all practical purposes, the corrections B_c and F_c are of negligible importance. Therefore, as a sufficient approximation, the menis-

cus contours of the various pores, arranged in a regular array, may be considered as lying in the equatorial planes of the spheres.

Accordingly in Eq. (2), $\int \sigma \cos\theta ds$ may be replaced by σp and one obtains for the maximum height of capillary rise

$$h = \frac{\sigma p}{\rho g a} . \tag{11}$$

This relation should give the maximum capillary rise for a system of uniform spheres in regular array. Uniform spheres packed in hexagonal array have a porosity $P = 0.26$ and in simple cubic array, $P = 0.48$. Experimentally

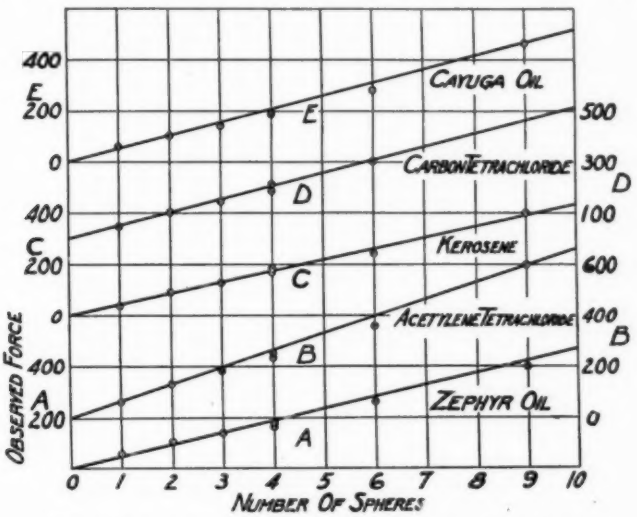

Fig. 4. Force required to withdraw spheres from liquid.

one can secure porosities in the region 0.30 to 0.45 while a porosity 0.38–0.40 is most readily obtained by simply pouring spheres into a large container. Under practical conditions therefore, an actual packing of spheres is far from an orderly array.

It is accordingly necessary to consider a typical minimum cell opening obtained by a statistical analysis. Slichter[3] in his studies of the flow of liquid through sands has employed a rhombohedral type of packing which may be skewed to represent in the limits either close hexagonal or simple cubic piling. In the manner used by Slichter, however, the model is not satisfactory either for capillary or flow problems. The writers[4] showed experimentally that a mixture of close hexagonal and simple cubic pilings in a proportion to give the required porosity suffices for the representation of the average number of contacts per sphere. This conception can be extended to the capillary problem with a fair degree of satisfaction, but the method outlined below was adopted in view of its greater simplicity. The use of any regular system of piling might

[3] Slichter, 19 Annual Report U.S.G.S. 1897–8, Pt. 2, pp. 295–384.

[4] Phys. Rev. **5**, 1271–4 (1929).

appear in contradiction with the statement made earlier that all geometrical planes passed through an extended quantity of sand have equivalent pore areas and perimeters. Actually, however, even with a "regular" piling, this condition is approximated because of random small distortions, axial rotations, and warpings of the systematic scheme. The meniscus follows closely the small distortions of the lattice so that the problem may be treated as though the arrangement were ideal.

Purely for statistical purposes the sand may be represented as packed in hexagonal array with a spacing of $2r+d$ between grain centers, where d is adjusted to the observed porosity. The number of spheres per cm^3 is $(2)^{1/2}(2r+d)^3$ and the total sphere volume is this quantity multiplied by $4\pi^3/3$. Hence for a porosity P,

$$(2r + d)^3 = \frac{4(2)^{1/2}\pi r^3}{3(1 - P)}. \tag{12}$$

Several types of minimum pore opening are present in such a hexagonal array, illustrated under the letters T, S and R of Fig. 5. It is readily seen

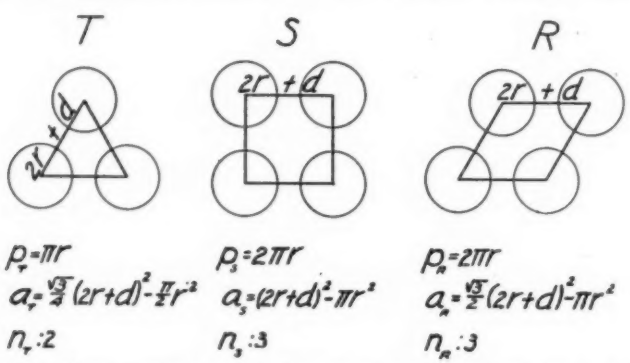

Fig. 5. Characteristic cells in hexagonal packing of spacing $2r+d$.

from an examination of a unit element, that these cells occur throughout the volume in a relative frequency

$$n_T : n_S : n_R = 2:3:3. \tag{13}$$

We shall make the reasonable assumption that in the capillary surface these cells are present with the same frequency distribution. The mean value of pr/a is accordingly the weighted sum of the perimeters for the three cells divided by the weighted sum of the pore openings.

$$\frac{\overline{pr}}{a} = \frac{2p_T + 3p_S + 3p_R}{2a_T + 3a_S + 3a_R}. \tag{14}$$

The values of p and a are tabulated under Fig. 5. Using these in conjunction with Eqs. (12) and (14) one obtains for the final capillary equation:

$$\max \frac{pr}{a} = \frac{\rho g h r}{\sigma} = \frac{2}{[0.9590/(1 - P)^{2/3}] - 1}. \tag{15}$$

This relation gives the maximum rise h for spheres of any uniform size packed in any observed porosity. In the experimental work it is more convenient to consider the dimensionless quantity pr/a as a function of the porosity since this is independent of the grain sizes and of the surface tensions and densities of the liquids employed.

The maximum rise h was determined by allowing the liquid to fall very slowly in a specially designed vessel containing packed spheres. The spheres employed were glass pearls of radius 0.0316 cm and Ottawa sands of radii 0.0443 cm and 0.0135 cm. The materials were carefully graded by screen separations and the radii were determined by direct measurement under a micrometer microscope. The pearls, produced by atomization of molten glass, were closely spherical while the sands were selected for the best degree of

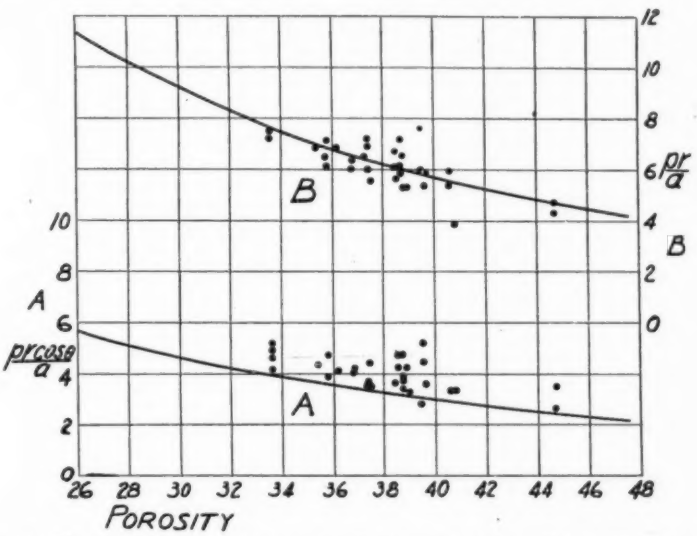

Fig. 6. The quantity pr/a for maximum rise and $(pr \cos \theta)/a$ for minimum rise as a function of the porosity.

sphericity obtainable. Porosities were measured by observing the volume occupied by a given weight of spheres of known density. Liquids employed varied widely with surface tension and density as follows: lubricating and crude oil, carbon tetrachloride, acetylene tetrachloride, toluene, benzene, xylene and water. The experimentally determined values of $\rho ghr/\sigma$ $(=pr/a)$ are shown as circles in the upper graph of Fig. 6, plotted against observed porosities. While the various data are scattered, there were found no systematic deviations of $\rho ghr/\sigma$ with ρ, r, and σ separately, and the controlling error appears to lie in the porosity measurements. Hence the individual data are not differentiated in the graph. The smooth curve through the observations is drawn according to Eq. (15) and represents the data for maximum rise within the probable error of the measurements.

In all cases a minimum rise was also observed. This was determined by allowing the liquid to rise slowly, in a previously wetted sand, until equi-

librium was reached. Observed values of $\rho ghr/\sigma$ ($=pr\cos\theta/a$) for the minimum rise as a function of the observed porosity, are plotted in the lower part of Fig. 6. It is simple to derive a relation for the minimum rise in a regularly arranged system. This is determined not by a plane through sphere centers but by a plane in which $pr\cos\theta/a$ is a minimum. For example in a close hexagonal packing of porosity 0.26, oriented with the rhombohedral faces horizontal the minimum value of $pr\cos\theta/a$ is 2.38. With the square faces horizontal the corresponding value is 3.88. We have been unable as yet however to devise a model of the typical cell which is consistent with the theory for the maximum rise and which enables the computation of the minimum rise at various porosities. It is of interest that the minimum rise is slightly greater than that computed by considering the sand as a cylindrical capillary.

Under such an assumption the value of $p\cos\theta$ per cm^2 is equal to the average value of $p\cos\theta$ for a sphere, multiplied by the number of spheres per cm^2 intersecting a plane passed at random through the volume. The value of a per cm^2 is equal to the porosity P. Hence, per cm^2

$$\left(\frac{\overline{pr\cos\theta}}{a}\right) = \frac{\Sigma pr\cos\theta}{\Sigma a} = \frac{2(1-P)r}{P} = \frac{\rho ghr}{\sigma}. \tag{16}$$

This relation is plotted as the lower curve of Fig. 6 and is only slightly below the observed data but we are unable to justify such a procedure.

In conclusion we desire to thank Dr. Morris Muskat for many helpful suggestions.

FLOW OF GAS THROUGH POROUS MATERIALS

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Abstract

An experimental study has been made to establish quantitatively the characteristics of and laws governing the flow of gases through consolidated and unconsolidated porous materials of fine texture. Experiments were performed with columns of glass beads, homogeneous and heterogeneous unconsolidated sands, as well as with samples of actual sandstones. With the sandstones linear flows were made both perpendicular and parallel to the bedding plane. Radial flows through annular sections of the sandstone were also made. In all cases it was found that the gradient of the squares of the pressures is proportional to a power of the mass velocity. In the various cases the exponent was found to lie between the limits 1 and 2 corresponding respectively to completely viscous and completely turbulent flow in straight cylindrical tubes. For a given sand however it remains fairly constant over a considerable range in the mass velocity. On the basis of these results a theory was developed for the production from and pressure decline in a closed sand reservoir of uniform thickness producing into a well under conditions of radial two-dimensional flow. The theoretical predictions check qualitatively with the limited field data that are available.

ALTHOUGH there have been several investigations[1] of gas flow through porous media no conclusive results seem to have been obtained for the flow through finely porous materials or for flows conducted at high pressures. As an accurate knowledge of the laws of flow under the above conditions is essential for the understanding of the production from an underground gas reservoir, the following investigation was undertaken.

I. Experiments on Unconsolidated Sands

Fine glass beads 0.632 mm in diameter were employed for the first series of measurements since it was felt desirable to use a material of uniform grain size. These beads were poured without shaking into the flow tube which throughout the experiments was held in a vertical position. This tube was of steel, 3.17 cm inside diameter and 92 cm long, and was mounted top and bottom with steel caps to the sides of which were fitted manometer connections for the measurement of inflow and outflow pressures. There were three additional manometer connections at approximately equal intervals along the tube as shown in Fig. 1. The glass beads were supported on a brass screen. This arrangement of manometers was to enable us to determine the nature of the pressure distribution along the sand column. Between the outflow end of the tube and the meter, a valve was placed so that any desired back pressure could be maintained on the tube. The two caps were also provided with thermometers in order to determine whether or not the flow was isothermal.

[1] C. C. Furnas, Flow of Gases Through Beds of Broken Solids, U. S. Bureau of Mines. Bull. 307. (1929). F. H. King, Nineteenth Annual Report, U. S. Geological Survey, (1897–98),

Flow measurements were made for various total pressure drops keeping the outflow pressure fixed. The time required for a given volume to flow through the tube at a given pressure drop was measure by a stopwatch. For low pressures, water, for higher pressures, mercury manometers were used, the latter being capable of reading to 45 pounds above atmosphere.

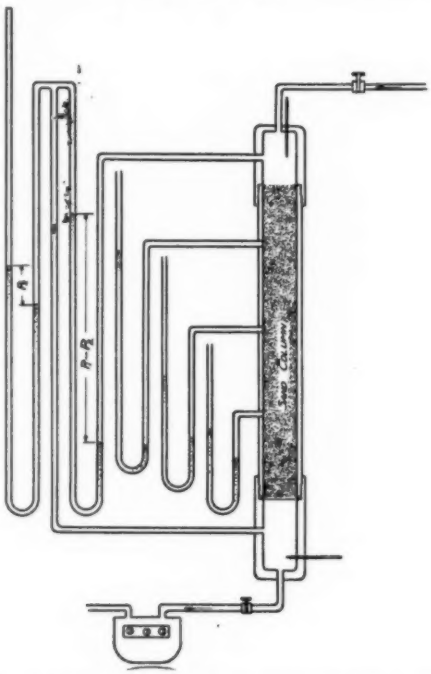

Fig. 1. Flow apparatus for unconsolidated sand.

Representative data for flow through glass beads are plotted in Fig. 2. In these plots ρv is the mass velocity, ρ being the density, and v the "effective velocity" of the gas in centimeters per second, obtained by dividing the volume of outflow per second by the cross-sectional area of the tube. The thermometers placed in the top and bottom of the tube indicated that the flow was always isothermal.

As will be shown in Section (III) the general law of flow of a gas through a sand may be expressed as:

$$\delta p^2 = k(\rho v)^n,$$

where k is the permeability constant of the sand and gas and n is a constant essentially determining the nature of the flow, varying between the limits 1 and 2, corresponding to completely viscous and completely turbulent flow respectively.

In order to obtain the value of n the data were plotted on log log paper as shown in Fig. 4, curve 1. From this the value of n is found to be practically 1.5; hence the equation for the flow through the glass beads may be written as:

$$\delta p^2 = k(\rho v)^{3/2}.$$

This equation will be discussed in detail in a later section. Among the factors which will affect the value of this exponent n, are porosity, grain size, and in a consolidated sand, the bond content will also be of considerable importance.

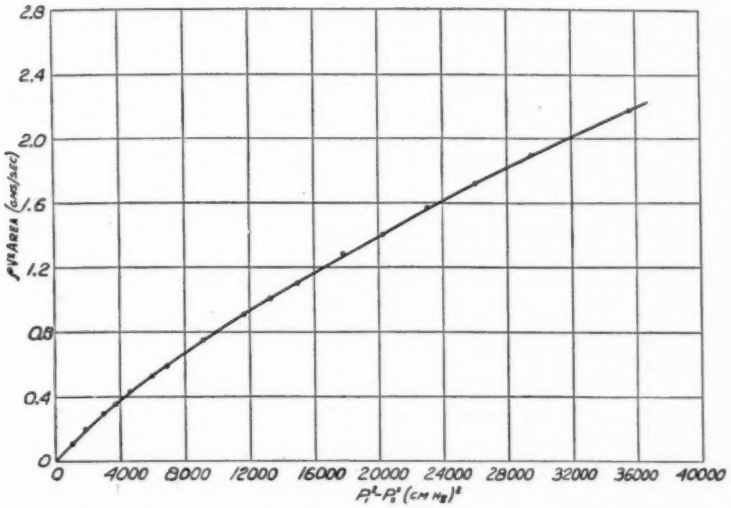

Fig. 2. Flow of air through glass beads at pressures up to 69 pounds absolute.

In order to note the direction of the variation of n with grain size, the flow tube was filled with graded sand (60–65 mesh). The porosity of this sand in the flow tube was 43.2 percent. Typical data from this run are plotted in Fig.

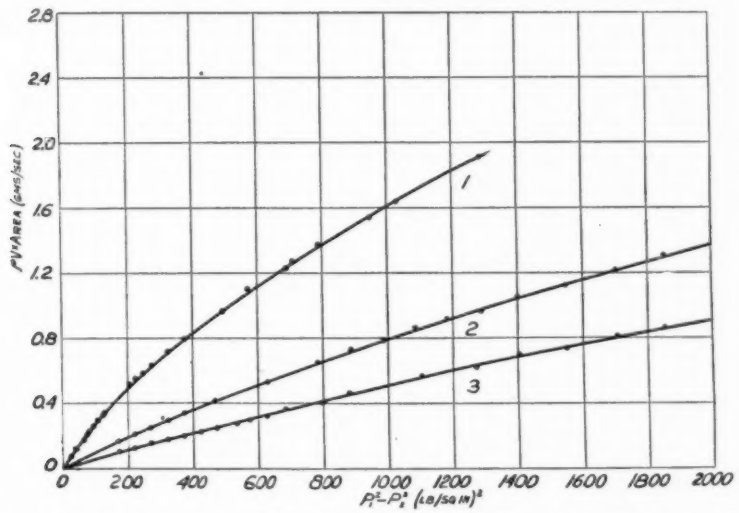

Fig. 3. Flow of air through unconsolidated media. Curve 1, Flow through glass beads. Curve 2, Flow through 60–65 mesh sand. Curve 3, Flow through heterogeneous sand.

3, curve 2. For the lower rates of flow in this and subsequent experiments, the volume flowing was determined by collection by displacement over water.

Data were also taken for the flow of air through heterogeneous sand of

porosity approximately 42 percent in place in the flow tube. These data are plotted in Fig. 3, curve 3. Curves 2 and 3, Fig. 4 may for practical purposes

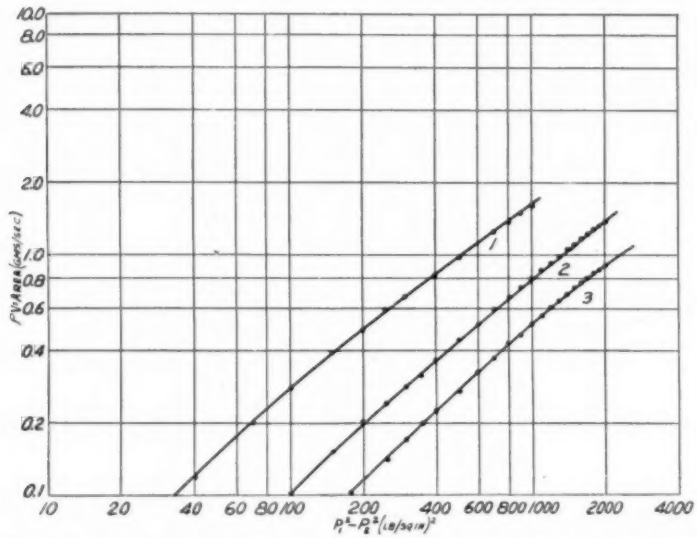

Fig. 4. Log-log curves of flow through unconsolidated media for determination of n. Curve 1, Flow through glass beads. Curve 2, Flow through 60–65 mesh sand. Curve 3, Flow through heterogeneous sand.

be assumed to be straight lines so that the exponent n may easily be obtained from them.

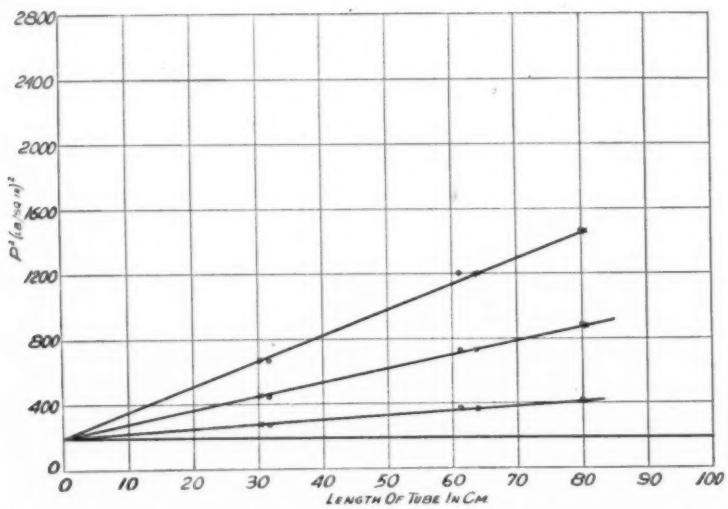

Fig. 5. Pressure distribution along flow tube at three different pressure gradients. (Material, heterogeneous sand.) Tube length is measured from the outflow end. Horizontal line indicates (atmospheric pressure)^2.

The intermediate pressures indicate as shown in Fig. 5, that p^2 is linear with respect to the distance along the tube to a high degree of accuracy. It

will be noted that the departure from linearity shown by the observed pressures is eliminated by assuming an error in location of the pressure orifices. This is reasonable since the effective location is not exactly that obtained by direct measurement of the locations of the rather large orifices. (See points marked x on the curves.) Slight differences in packing would produce a similar result but the above explanation is the most probable one.

II. Experiments on Consolidated Sands

In order to determine how closely the flow through unconsolidated media compared with flow through an actual sandstone, a large block of uniform sandstone with a porosity of 19.65 percent was procured and samples on which to make flow measurements, were drilled from this large block.

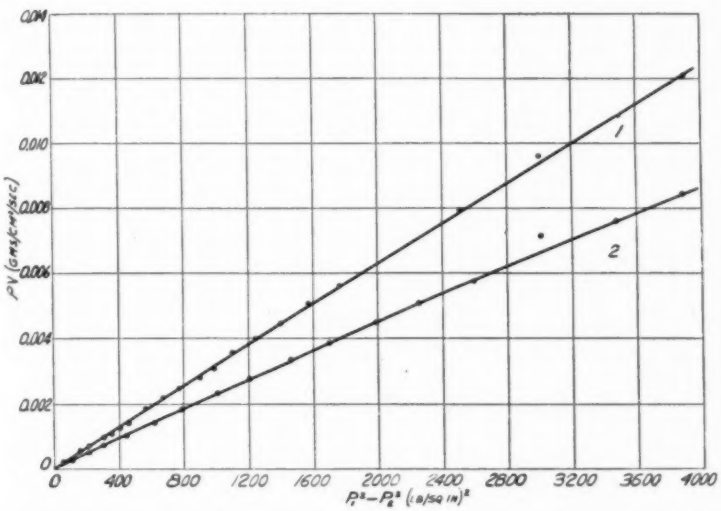

Fig. 6. Linear flow of air through sandstone. Curve 1, Flow parallel to bedding plane. Curve 2, Flow perpendicular to bedding plane.

Three different types of sample were prepared. Two of them were simply solid cores cut from the block, one parallel, and the other perpendicular to the bedding plane. The third sample was a hollow cylinder cut with the axis perpendicular to the bedding plane for measurement of radial flow.

The first two samples were each 4.8 cm in diameter and 12.7 cm long. The sides of these were coated with tar and they were sealed in iron pipe nipples 12 inches long. Manometers were connected to the pipe above and below the sandstone samples. The flow measurements were made in exactly the same manner as with the unconsolidated sands. The data for the flow parallel to the bedding plane are given in Fig. 6, curve 1 while the results of flow perpendicular to the bedding plane are contained in curve 2, of the same figure. It will be noted that the flow perpendicular to the bedding plane is about 70 percent as great for the same pressure drop as is the flow parallel to the bedding. This is a very uniform sandstone so that it may be expected that others less uniform will show a greater difference in permeability in the two direc-

tions. It will be noted that these two curves are so nearly straight that when plotted on double log paper, as in Fig. 7, the flow appears to deviate only very slightly from viscous flow over the region measured. The ordinates ρv in all

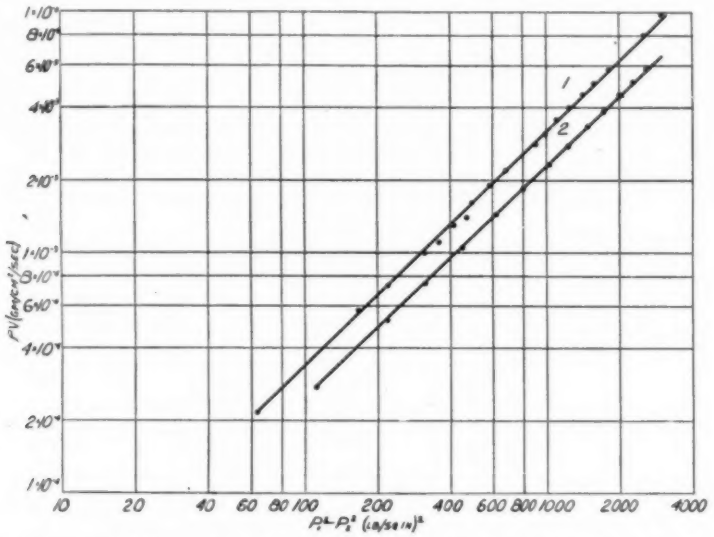

Fig. 7. Log-log curves of linear flow through sandstone for determination of n. Curve 1, Flow parallel to bedding plane. Curve 2, Flow perpendicular to bedding plane.

these curves for flow through consolidated media are mass velocity or the mass outflow per second per cm^2 of area, thus enabling a direct comparison between the curves for linear and for radial flow.

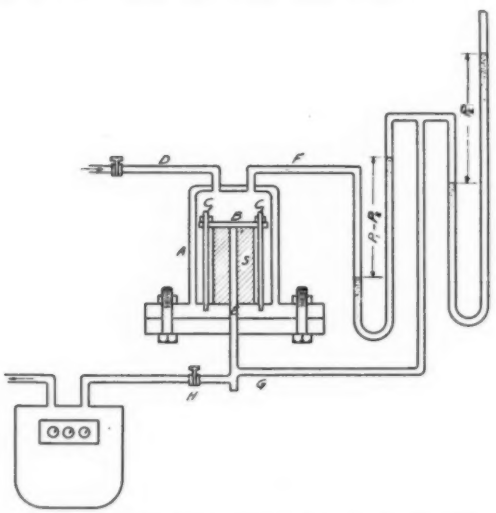

Fig. 8. Apparatus for radial flow measurements.

For the radial flow experiments the hollow cylinder of sandstone was held in a steel container as shown diagrammatically in Fig. 8. A is the steel container, B a metal plate to seal the end of the sample, C are rods by means of

which the sample and plate are held in position. F and G are manometer connections. The back pressure on the system is maintained by means of valve H. Air entered through D, flowed radially through the sample from the outside in and thence through E to the meter. As was mentioned above, the sample was so cut that the flow was parallel to the bedding plane.

For the first flows, the diameter of the central hole through the sample was made as small as possible in order to get the maximum effect of the radial flow. The sample was 10 cm long, 7.8 cm outside diameter and the central hole was 0.8 cm in diameter. This sample was cut from the large block of sandstone in a drill press by revolving iron cylinders using water and coarse carborundum powder as an abrasive.

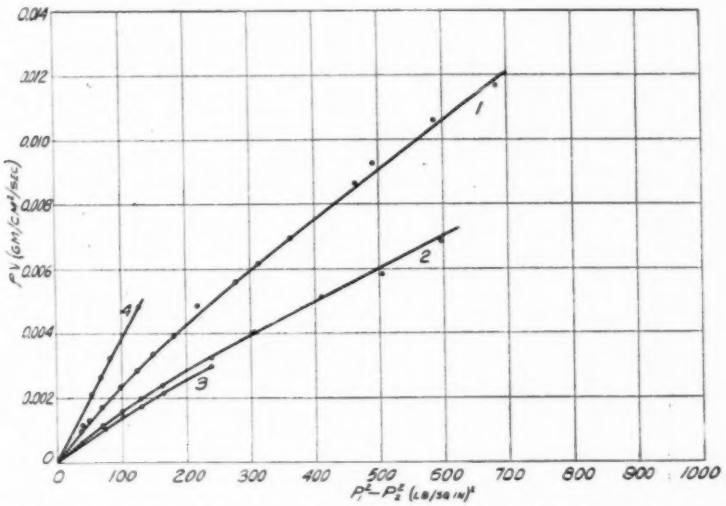

Fig. 9. Radial flow through sandstone. Curve 1, Initial flow. Curve 2, Flow after lightly scraping out inside surface. Curve 3, Flow after enlarging diameter of inside hole 1 mm. Curve 4, Flow after re-drilling inside hole.

The data for the first runs are plotted in Fig. 9, curve 1. It is to be noted that at a mass velocity of about 0.002 gm/cm²/sec there appears to be a fairly well-defined deviation from linearity indicating that the beginning of a transition from viscous to turbulent flow occurs in this region. This checks the linear flow measurements in which a much less pronounced trend may be noticed in the same region.

The theoretically derived equation for the viscous radial flow of a gas is, as is shown in Section (III) Eq. (12),

$$Q = \frac{2\pi k \delta p^2}{\log r_0/r_w},$$

where Q is the mass outflow in unit time per unit thickness of sand; δp^2 is the difference in the squares of the pressures at the exterior and interior surfaces of the annulus defined respectively by the radii r_0 and r_w. Since the two sandstones used for radial and linear flows were cut from the same block, the per-

meability as determined from linear flow data should be approximately the same as that determined from radial flow. However the k_{radial} when calculated was found to be practically one half the k_{linear}. This result was very puzzling at first but it was finally decided that in the process of drilling the central hole in the radial flow specimen, the fine drilling mud might have plugged up the surface pores sufficiently to give the observed reduction in permeability.

The central hole was then very carefully scraped out with a file, care being taken that the radius of the hole was not noticeably increased. The results for the flow after this treatment appear in curve 2, Fig. 9. As is seen, from the curve, the permeability was still further reduced by this slight scraping.

With the thought in mind that possibly this layer of reduced permeability was a very thin one, the hole was again scraped out sufficiently to enlarge its diameter by 1 mm. However this still further reduced the permeability as may be seen from curve 3, Fig. 9.

It was decided that since the sample was dry before drilling, capillarity had forced drilling mud into the pores, forming a shell of low permeability and appreciable thickness about the inner hole. To reduce this effect as much as possible the sample was soaked in water for several days and the hole was then enlarged by drilling while the specimen was kept saturated with water.

After this treatment and subsequent thorough drying, flow measurements on the sample resulted in the data plotted in curve 4, Fig. 9. The permeability factor increased to a value very near that obtained for linear flow.

TABLE I. *Permeabilities and porosities. Permeability in gm/sq cm/sec/(atmos.²/cm)$^{1/n}$.*

Material	Permeability	Porosity
Glass Beads	6.02–1.31	33.8%
60–65 Mesh Sand	1.04	44.0%
Heterogeneous Sand	0.811	42.0%
Sandstone parallel to bedding	0.00892	19.65%
Sandstone perpendicular to bedding	0.00638	19.65%
Sandstone radial flow	0.00715	19.65%

Table I shows the permeabilities of the various materials used as calculated from the data obtained. In calculating these values the exponent used was obtained from the curves as plotted on log-log graph paper in Figs. 4 and 7. The permeability k is obtained by solving

$$\rho v = k\left(\frac{\delta p^2}{\delta s}\right)^{1/n}$$

for k and substituting the values of ρv and $\delta p^2/\delta s$ obtained from the curves; n is determined by the slope of the line at the point under consideration and its value indicates the nature of the flow. For example k was calculated for three separate points on curve 1, Fig. 4, at $\rho v = 0.174$ gm/sec, 0.3 gm/sec and 1.12 gm/sec. The first two points are in a region where the flow is practically viscous. $n = 1$ and k is practically the same for both points, about 6.02. For the third point where the flow is much more turbulent (n is 1.5) k is found to be

1.31 thus showing the dependence of k upon the exponent n. The importance of this will be discussed more fully later on.

Relation of exponent n to sand characteristics.

The fact that the value of the exponent n is a function of porosity, grain size and bond content has already been mentioned. In general it may be said the value of n appears to decrease with decrease in grain size although no quantitative relationships have been determined, or even attempted, since it is apparent that such relations would be quite complex. Further, in our case we are not primarily interested in this relation since to measure porosity, "effective" grain size, degree of cementation, etc., in an actual sand is a far greater task involving greater error than a direct measurement of the permeability factor k for a sample of the sand in question. Thus for practical reasons, no attempt has been made to study this phase of the problem.

III. Discussion and Applications

Dimensional considerations.

From a theoretical point of view the motion of a gas through the tortuous paths in a column of sand is far too complicated to permit an analysis by means of the classical theory of hydrodynamics. This fact has long been recognized in the case of the flow of liquids through porous media, where the empirically established law (D'Arcy law) that the velocity is proportional to the pressure gradient has been taken as the basis of the hydrodynamics rather than the Stokes-Navier equations. The problem of the flow of compressible fluids would present a still greater complexity so that an analysis by the classical hydrodynamics is entirely out of the question.

However, a dimensional analysis of the problem can be made very readily, and this suffices to indicate the manner in which the important variables of the problem should enter into the final expression for the pressure drop along the sand column. In fact, the usual dimensional considerations show that for any fluid the pressures and velocities are related by an equation of the form:

$$\delta p = \text{const}\left(\frac{\mu^2}{\rho d^2}\right) F\left(\frac{dv\rho}{\mu}\right)\Phi\left(\frac{\delta s}{d}\right), \tag{1}$$

where δp is the pressure drop over a length of sand δs, v is the average velocity of the fluid along δs, and d is a length representing the average size of pore openings.

We may suppose that the unknown functions F and Φ may be expressed in terms of powers of their arguments. Thus Eq. (1) would become

$$\delta p = \text{const}\left(\frac{dv\rho}{\mu}\right)^n \left(\frac{\delta s}{d}\right)^m \left(\frac{\mu^2}{\rho d^2}\right), \tag{1a}$$

where the values of the constants n and m and the constant coefficient, as well as the number of terms of the above type in the expression for δp, must be determined by experiment.

For gas flow the density ρ will vary with the pressure p and the temperature T of the system, the exact relation being determined by the thermodynamic character of the flow. As pointed out in Section I. thermometers at the ends of the steel flow tube showed no temperature differences. Although it is certainly possible that this may have been due to the highly conducting character of the steel sheath, it does not seem unreasonable to suppose that generally, except at very high velocities of flow; the expansion of the gas will be approximately isothermal. For definiteness it has been explicitly assumed that the gas expansion at least under the experiments conducted is a strictly isothermal expansion of an ideal gas. This necessarily implies that the density ρ is proportional to the pressure, i.e.

$$\rho = cp; c = \frac{w}{RT}, \tag{2}$$

where p is the average pressure over the element δs, w is the molecular weight of the gas, T, its absolute temperature, and R is the gas constant per mole. When this is put into Eq. (1a) it may be written as:

$$\delta p^2 = \text{const} \left(\frac{dv\rho}{\mu}\right)^n \left(\frac{\delta s}{d}\right)^m, \tag{3}$$

where now the constant has taken in the factor $2\mu^2/cd^2$. It is of interest to note from this equation that the fundamental dependent variable for gas flow is p^2, or in particular, δp^2, whereas in the problem of liquid flow it is readily seen from Eq. (1a) that the corresponding dependent variable is δp.

As the experiments described in Section I have shown that p^2 varies linearly with the distance along the cylindrical column of sand it follows at once from Eq. (3) that the exponent $m = 1$; that is what we should require *a priori* if the results of the experiments on the cylindrical column are to be independent of the shape and size of this vessel and be adaptable to differential sand elements. Since for a given sand column δs and d are fixed we have therefore been justified in including the factor $(\delta s/d)^m$, as well as $(d/\mu)^n$ into the constant coefficient and in considering, in the cylindrical tube experiments δp^2 as a function only of ρv, with the understanding that on rewriting the results in the form

$$\frac{\partial p^2}{\partial s} = k(\rho v)^n \tag{4}$$

it will be the differential equation governing the flow in a vessel of any shape whatever. In particular, then since $\delta p^2/\delta s$ and ρv are constant along the length of a cylindrical tube it was quite sufficient in determining the "law of flow" to study $\delta p^2 = p_1^2 - p_2^2$ taken at the ends of the sand column, as a function of the mass velocity, ρv.

The values of n, determined in this way, can be given some physical significance if the general law of flow expressed by Eq. (1a) is applied to the flow through sand free cylindrical tubes. Thus it may be shown from the

classical hydrodynamics that for viscous streamline isothermal gas flow through a cylindrical tube,

$$\delta p^2 = \frac{16\mu}{cd^2}(\rho v)\delta s, \tag{5}$$

where v in this expression is the average velocity across the section of the tube, and d is its radius. Obviously this corresponds exactly to Eq. (1a) with m and n set equal to 1. It therefore appears that the flow through sands for which $n=1$, is equivalent to the viscous flow of the gas through a sand free cylindrical tube, and hence may be called "viscous flow."

On the other hand, from experiments on the flow of gases through pipes, it is found that for high velocities, and in particular when the Reynold's number, $dv\rho/\mu$, exceeds a definite critical value the nature of the flow changes from a type obeying Eq. (5) to a new type described by an equation of the form:

$$\delta p^2 = \text{const } (\rho v)^2. \tag{6}$$

This flow, called "turbulent," clearly corresponds to the value of $n=2$ in Eq. (1a). States of flow in pipes corresponding to values of $n<1$ or >2 are not known.

This shows that the laws of flow for which $n>1$, such as were established in Sections I and II represent partially turbulent states of flow. That n does not change abruptly from the value 1 to a value 2, in the flow through a sand but that instead it takes on intermediate values is clearly due to the fact that the tortuous and capillary nature of the paths through which the gas can flow do not permit a sudden and complete change of the stream lines into a system of eddies, characteristic of turbulent pipe flow.

The same point of view permits us even to anticipate the result demonstrated in Section I and plotted in Fig. 3, e.g.: as the porosity and average grain diameters of the sand decreased the value of n, i.e. the degree of turbulence, for corresponding values of δp^2, decreased also. Although it is certainly true that at sufficiently high velocities and pressure gradients eddies will form in the sand pores, it seems equally certain that the capillary size and irregularities in the path through which these eddies must travel will as a whole tend to break these eddies or at least inhibit their development into a steady and continuous stream through the sand body.

It has already been noted that the laws of flow experimentally derived may be applied to differential elements of a sand body. Since the log log plots of the data in Section I for the finer sands have shown that over considerable ranges of the mass velocity, the law of flow is of the form of Eq. (3) with only a single term on the right hand side, we shall hereafter confine our discussions only to such simplified forms of Eq. (3).

It will now be convenient to invert Eq. (4) and rewrite it as:

$$\rho v = k\left(\frac{\partial p^2}{\partial s}\right)^{1/n}. \tag{4a}$$

This equation is really a vector equation and gives the vector mass velocity along the line element ds, the direction of flow being of course in the direction of decreasing p^2. It may readily be shown that its component equations for general three dimensional flow are given by

$$\rho v_x = \frac{k \dfrac{\partial p^2}{\partial x}}{\left(\dfrac{\partial p^2}{\partial s}\right)^{(n-1)/n}}; \tag{4b}$$

etc. where

$$\frac{\partial p^2}{\partial s} = \left[\left(\frac{\partial p^2}{\partial x}\right)^2 + \left(\frac{\partial p^2}{\partial y}\right)^2 + \left(\frac{\partial p^2}{\partial z}\right)^2\right]^{1/2}. \tag{4c}$$

With regard to the above differential expressions it should be understood that an infinitesimal increment as ds really implies a distance extending over a large number of sand grains and that p and v at (x, y, z) represent averages over many pores immediately surrounding (x, y, z).

The production decline from a gas reservoir.

An interesting and practical problem in which the study of the laws of gas flow in porous media finds direct application is that of the decline in production and pressure in a gas sand formation. Before considering such problems we must introduce the time parameter into our equations. This may be done as follows:

From the physical requirement that no mass is created or destroyed within the sand formation we get the following well-known "equation of continuity":

$$\operatorname{div}(\rho \bar{v}) = -\frac{f \partial \rho}{\partial t} \tag{7}$$

where the left hand side denotes the net mass outflow of gas from unit volume in unit time, and the right side denotes the equivalent value of the density change in the time unit, f being the porosity of the sand. The $\rho\bar{v}$ in this expression represents the vector mass velocity. Assuming isothermal flow in a homogeneous sand and applying Eqs. (2) and (4a), Eq. (7) becomes:

$$\operatorname{div}\left[\left(\frac{\partial p^2}{\partial s}\right)^{1/n}\right] = -\frac{fc}{k}\frac{\partial p}{\partial t}. \tag{8}$$

For steady states we get

$$\operatorname{div}\left[\left(\frac{\partial p^2}{\partial s}\right)^{1/n}\right] = 0. \tag{8a}$$

The differential Eq. (8) governs the change of the pressure with time at any point in the sand formation corresponding to any preassigned boundary and initial conditions. Unfortunately, however, as the equation is obviously non-linear there are no satisfactory methods known for solving it exactly.

The following physical approximation has therefore been introduced into the problem. Instead of considering the problem of gas flow from a sand formation as a continuously varying system governed by Eq. (8) we shall assume that the actual behavior of the system is equivalent to a continuous succession of steady states each governed by Eq. (8a). This is essentially equivalent to the assumption that the velocity of the flow is negligible compared to the velocity of propagation of disturbances through the sand formation.

Taking for simplicity the case of radial horizontal flow we may make the following model of a gas producing formation. Let the well, Fig. 10, defined

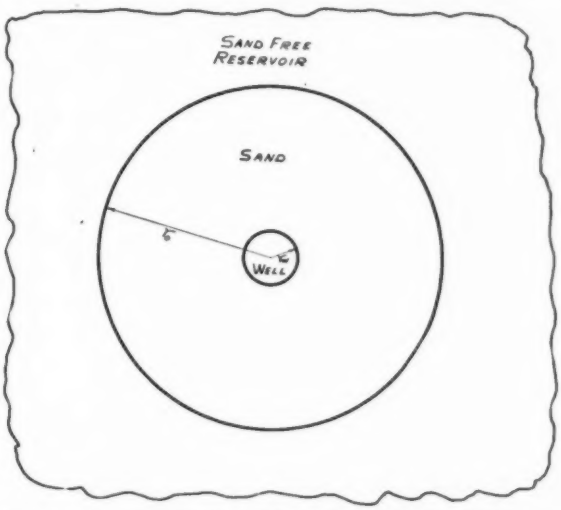

Fig. 10. Diagrammatic model of a producing formation.

by the circle of radius r_w, be approximately at the center of the sand formation which it is draining. Since the pressure gradients at large distances from the well are very small we shall neglect them entirely and suppose that beyond a distance r_0 there is no sand at all and that the region between r_0 and the boundary of the formation is simply a sand free reservoir supplying the gas flowing into the inner "sand disk" and thence into the well at r_w. Further. we shall suppose that the flow through the inner sand disk is governed by Eq. (8a) with the continuously varying boundary pressure at $r = r_0$ equal to the pressure in the reservoir, this latter declining due to the depletion of the gas as it flows out through the well.

We can now solve the problem without difficulty. For radial two-dimensional flow, Eq. (8a) takes the form:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\partial p^2}{\partial r}\right)^{1/n}\right] = 0, \tag{8d}$$

so that

$$p^2 = \frac{p_0^2 - p_w^2}{\dfrac{1}{r_0^{n-1}} - \dfrac{1}{r_w^{n-1}}}\left(\frac{1}{r^{n-1}} - \frac{1}{r_w^{n-1}}\right) + p_w^2;\ n \neq 1, \tag{9}$$

$$p^2 = \frac{p_0^2 - p_w^2}{\log r_0/r_w} \log \frac{r}{r_w} + p_w^2; \; n = 1, \tag{10}$$

where p_w and p_0 are the pressures at r_w and r_0, respectively. These give the pressure distributions in the "sand disk." The instantaneous mass outflows through the well are then obtained on applying Eq. (4a), with s changed to r, to these expressions. The values so found are:

$$Q = 2\pi rb\rho v = 2\pi kb\left[\frac{(p_0^2 - p_w^2)(n-1)}{\dfrac{1}{r_w^{n-1}} - \dfrac{1}{r_0^{n-1}}}\right]^{1/n} n \neq 1, \tag{11}$$

$$= 2\pi kb \frac{p_0^2 - p_w^2}{\log r_0/r_w}; \; n = 1, \tag{12}$$

where b is the sand thickness.

We now suppose, according to the model just described, that the pressure p_0 represents the instantaneous pressure in the sand "reservoir," and for simplicity that p_w is maintained constant. If V is the volume of the reservoir, then

$$Q = -\frac{wV}{RT}\frac{dp_0}{dt}, \tag{13}$$

where w is the molecular weight of the gas, T its temperature and R is the gas constant per mol. We therefore get, on introducing the dimensionless variable $x = p_0/p_w$

$$\frac{dx}{dt} = -C_n(x^2 - 1)^{1/n}; \tag{14}$$

$$C_n = \frac{2\pi kbRTp_w^{2/n-1}}{wV}\left(\frac{n-1}{\dfrac{1}{r_w^{n-1}} - \dfrac{1}{r_0^{n-1}}}\right)^{1/n}; \; n \neq 1 \tag{14a}$$

$$C_1 = \frac{2\pi kbRTp_w}{wV \log r_0/r_w}; \; n = 1. \tag{14b}$$

The integrals of these may be expressed as:

$$C_n t = \int_x^{x_i} \frac{dx}{(x^2 - 1)^{1/n}}, \tag{14c}$$

where $x_i = p_i/p_w$, p_i being the initial or the "rock" pressure of the reservoir.

For general values of n between 1 and 2, Eq. (14c) may be integrated by expanding the integrand as follows:

$$\left.\begin{aligned}\int_x^{x_i} \frac{dx}{(x^2-1)^{1/n}} &= \sum_0^\infty c_{nj} \int_x^{x_i} \frac{dx}{x^{2j+2/n}} \\ &= \sum_0^\infty \frac{c_{nj}}{\left(2j + \dfrac{2}{n} - 1\right)}\left(\frac{1}{x^{2j+2/n-1}} - \frac{1}{x_i^{2j+2/n-1}}\right) \\ &= f_n(x) - f_n(x_i) = C_n t,\end{aligned}\right\} \tag{15}$$

where

$$c_{nj} = \frac{\Gamma\left(j + \frac{1}{n}\right)}{\Gamma\left(\frac{1}{n}\right)\Gamma(j+1)} = \frac{1}{jB\left(j, \frac{1}{n}\right)}, \tag{15a}$$

and

$$f_n(x) = \frac{1}{x^{2/n-1}} \sum_0^\infty \frac{c_{nj}}{\left(2j + \frac{2}{n} - 1\right)x^{2j}}. \tag{15b}$$

For $n=1$, $3/2$ and 2, one may derive either from the series or by direct integration the following closed expressions:

$$\left.\begin{aligned} &n = 1, \text{ viscous flow,} \\ &x = ctnh(C_1 t + \gamma) = ctnh\, y \\ &y = C_1 t + \gamma;\ \gamma = ctnh^{-1} x_i \\ &n = 3/2, \text{ partially turbulent flow,} \end{aligned}\right\}. \tag{16}$$

$$x = \frac{2(3)^{3/4} snz\, dnz}{(1 - cnz)^2}, \tag{17}$$

where

$$\left.\begin{aligned} z &= 2(3)^{1/2}(\beta + \tfrac{1}{3}C_{3/2}t) \\ \beta &= p^{-1}[(x_i^2 - 1)^{1/3}] \end{aligned}\right\}, \tag{17a}$$

p being the Weierstrassian elliptic function, the modulus of the elliptic functions being given by: $k^2 = 0.933$.

$$\left.\begin{aligned} &n = 2, \text{ completely turbulent flow,} \\ &x = \cosh(\delta - C_2 t) = \cosh y, \\ &\delta = \cosh^{-1} x_i;\ y = \delta - C_2 t \end{aligned}\right\}. \tag{18}$$

where

The volumetric production decline curves may easily be obtained from the above expressions. If we denote by V_a the volumetric production per unit of time, p_a the atmospheric pressure, and m the mass of gas in the reservoir, then:

$$V_a = -\frac{RT}{wp_a}\frac{dm}{dt} = -\frac{Vp_w}{p_a}\frac{dx}{dt} \equiv -K\frac{dx}{dt}. \tag{19}$$

From this we can derive at once the following results:
For $n=1$:

$$\left.\begin{aligned} V_a &= KC_1 \operatorname{csch}^2 y \\ y &= C_1 t + \gamma \end{aligned}\right\}. \tag{20}$$

For $n=3/2$:

$$V_a = KC_{3/2}(x^2 - 1)^{2/3}, \tag{21}$$

where x is given by Eq. (17).

For $n=2$:

$$\left.\begin{aligned} V_a &= KC_2 \sinh y \\ y &= \delta - C_2 t \end{aligned}\right\}. \tag{22}$$

In order to discuss the problem of decline numerically it will be necessary to make a choice of the physical and dimensional constants that enter into our equations, in particular in the coefficient C_n of Eq. (14a). Perhaps the most significant of these is the value of k, the permeability constant. Up to the present we have carried along k as if it were an absolute constant of the sand and the gas flowing through it. However, this is not at all the case, although this fact will in no way change any of our equations. Fundamentally k is a function of n, which in turn depends on the *nature* of the flow. As explained on page 34 this follows from the fact that k is given by:

$$k = \frac{\rho v}{\left(\frac{\partial p^2}{\partial s}\right)^{1/n}}.$$

Hence if the slope n, of the log-log curves as plotted in Fig. 7 is not a constant, k will not be constant. It therefore follows that the permeability constant k must be associated with the nature of the flow as well as with the nature of the sand and gas. However as Fig. 7 shows, the slope is practically constant over a considerable range in the variation of the mass velocity. It will therefore suffice for our present purposes to consider n and hence k as actually constant, and consider cases of varying n separately as explained below.

As we wish, in the following, to illustrate only the order of magnitudes of the productions involved in the decline curves we shall suppose that the same point on the flow curve such as is given in curve 1, Fig. 6 refers simultaneously to flows with $n=1$, $3/2$ and 2. The point we shall choose is given by:

$$\rho v = 0.0033 \text{ gm/cm}^2\text{/sec}; \delta p^2 = 1000 \text{ lb.}^2\text{/in}^4.$$

Hence from Eq. (4a) and noting that in the experimental work $\delta s = 12.7$ cm

$$k_n = 0.0033\left(\frac{12.7}{4.63}\right)^{1/n}.$$

Taking the time unit as the day instead of the second and using cgs units throughout, we get then:

$$\left.\begin{aligned} k_1 &= 790 \text{ gm/day/cm}^2 \text{ (atm}^2\text{/cm)} \\ k_{3/2} &= 560 \text{ gm/day/cm}^2 \text{ (atm}^2\text{/cm)}^{2/3} \\ k_2 &= 473 \text{ gm/day/cm}^2 \text{ (atm}^2\text{/cm)}^{1/2} \end{aligned}\right\}. \tag{23}$$

We have now to choose the physical dimensions of the well and reservoir systems. For these we shall take the following:

$r_w =$ 15 cm $\sim$ 6″ = well radius.

$r_0 =$ 6000 cm $\sim$ 200′ = outer radius of sand disk.

$b =$ 300 cm $\sim$ 10′ = thickness of sand.

We shall suppose the sand free reservoir volume to be equivalent to that between r_0 and a circle at $r_1 = 30{,}000$ cm$\sim$1000′. The volume of the reservoir is then:

$$V = \pi b(r_1^2 - r_0^2) = 8.14 \times 10^{11} \text{ cm}^3 \cong 2.9 \times 10^7 \text{ cu ft}.$$

This means that if the initial rock pressure is about 1000 lb. the total production capacity would be about 2×10^9 cu. ft. If we assume that the gas is methane and the temperature of the system is 300°K we can compute the C_n from Eq. (14a). The values found are:

$$\left.\begin{aligned} C_1 &= 4.68 \times 10^{-4} p_w \\ C_{3/2} &= 3.20 \times 10^{-3} p_w^{1/3} \\ C_2 &= 6.53 \times 10^{-3} \end{aligned}\right\}. \tag{24}$$

For p_w we shall take the value 4 atmospheres$\sim$58.8 lb. Putting in this value in (24) and then the C_n in Eq. (16) and (18) we get the decline curves as plotted in Figs. 11 to 14 for the pressure and productions, for the values of $x_i = 10$ and $x_i = 20$, corresponding to initial rock pressures of 588 lb. and 1176 lb. respectively.

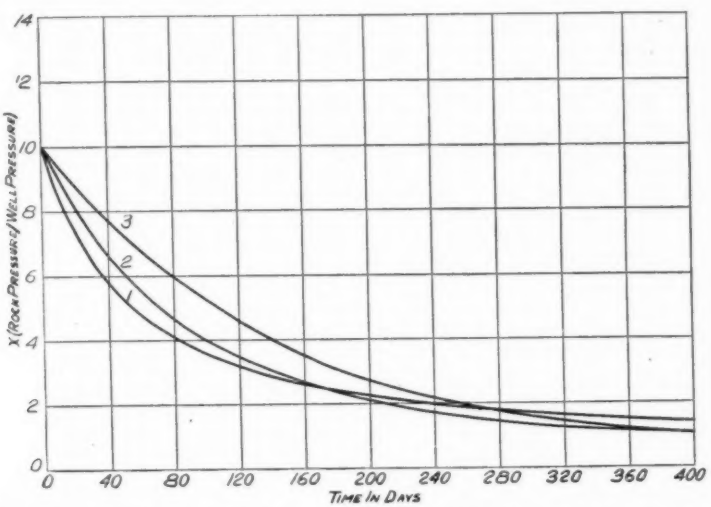

Fig. 11. Pressure decline for $x_i = 10$. Curve 1, $n = 1$, viscous flow. Curve 2, $n = 3/2$, partially turbulent flow. Curve 3, $n = 2$, completely turbulent flow.
$x_i =$ (initial rock pressure)/(well pressure)

We may note the following points of interest about these curves. Perhaps their most striking feature is the very rapid decline in pressures and production during the initial stages of the flow. Thus within a period of several months the production and pressures decline to small fractions of their initial

values. Although this may at first be surprising, it represents the actual situation as observed in the field for gas wells.

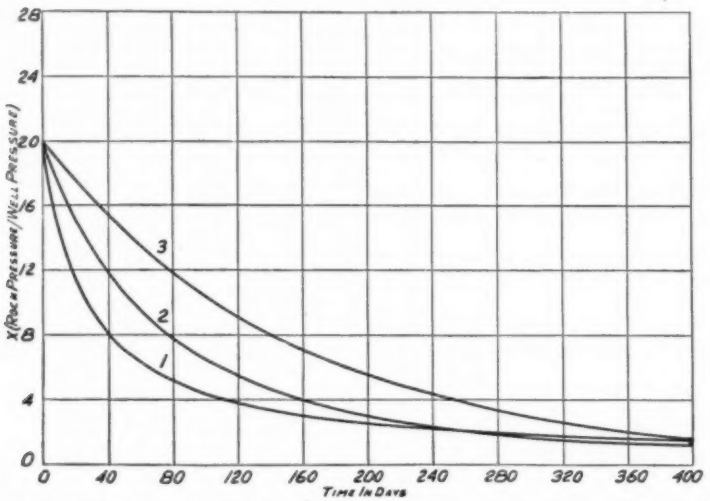

Fig. 12. Pressure decline for $x_i = 20$. Curve 1, $n = 1$, viscous flow. Curve 2, $n = 3/2$, partially turbulent flow. Curve 3, $n = 2$, completely turbulent flow.
x_i = (initial rock pressure)/(well pressure)

Further, this is particularly interesting in view of our physical approximation made at the beginning of the analysis involved in using the steady state Eq. (8a) instead of the rigorous Eq. (8). It is not difficult to see that the

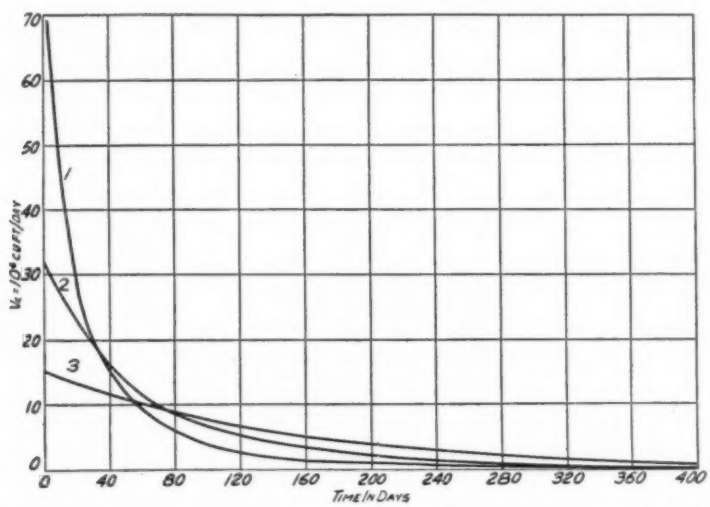

Fig. 13. Production decline for $x_i = 10$. Curve 1, $n = 1$, viscous flow. Curve 2, $n = 3/2$, partially turbulent flow. Curve 3, $n = 2$, completely turbulent flow.
x_i = (initial rock pressure)/(well pressure)

rigorous solution of Eq. (8) would necessarily result in a still higher initial production and consequent more rapid initial decline. This becomes clear

when we note that the initial production according to Eq. (8) would correspond to a pressure distribution which is almost constant up to the immediate vicinity of the well with extremely high gradients at the well face, whereas the solutions we have found, of Eq. (8a), correspond to pressure distributions of the type given by Eq. (9) with much lower gradients at the well face. Of course, after a time, the production rate according to Eq. (8) will be less than that which would be predicted by the solutions of Eq. (8a), but our initially high rates of production and decline must be considered as minimal values as compared to the values predicted by the rigorous Eq. (8).

Another point of interest about the general form of the decline curves is that the productions fall to 0 and the parameter x to the value 1 within a finite time for the cases of $n=3/2$ and $n=2$. This is not evident from the graphs due to the scales used in plotting, but Eqs. (17), (18), (21), and (22)

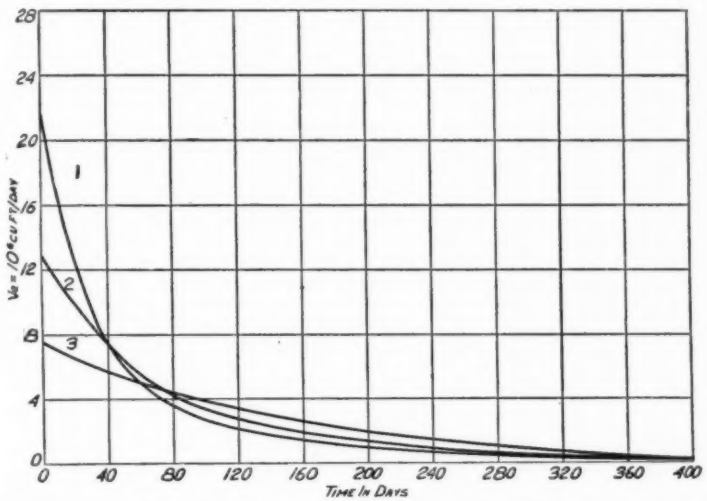

Fig. 14. Production decline for $x_i=20$. Curve 1, $n=1$, viscous flow. Curve 2, $n=3/2$, partially turbulent flow. Curve 3, $n=2$, completely turbulent flow.
x_i = (initial rock pressure)/(well pressure)

will show at once that this must be the case. It is readily verified by examining Eq. (14c) that this will also be true for any value of $n>1$ and that only if $n=1$ will it take an infinite time for the production to decline to 0. Physically we would expect any equilibrium phenomenon such as the pressure decline in a finite reservoir to take an infinite time for it to reach equilibrium.

However, the following considerations remove this difficulty that our solutions for $n>1$ imply only a finite time for the complete drainage of the gas reservoir. Strictly speaking, we are not at all justified in expecting the decline curves for $n>1$ to be valid over the whole life of a well. Certainly if there is any physical meaning to the value of the exponent n it cannot remain greater than 1 both during the initially high production rates, and the very low rates of production that must follow in the later life of the well. Although we can-

not predict definitely what n will be at the beginning of the decline, there can be no question but that it will fall to $n=1$ after the reservoir has been appreciably depleted. Hence when considering the decline curves for the last stages of the decline we must use the curves for $n=1$. These do lead to an infinite time for depletion and at the same time to a resolution of our difficulty.

These same considerations also bring out the fact that one should not expect any actual decline curve as observed in practice to be representable as a single analytical curve of the type we have derived from Eq. (8a). Rather, they will be composed of a number of such curves corresponding to gradually decreasing values of n, smoothly joined together, although for practical purposes one may synthesise them with only 2 or 3 values of n. In fact, we may even anticipate that in certain cases the production at a given time may not correspond to a uniform value of n but that instead there will be a variation in n through the formation, decreasing of course, as one recedes from the well. Such conditions would in general be very difficult to treat analytically. However, if we assume that over definite regions n remains uniform one may carry through the required calculations without difficulty.

In addition to this type of inhomogeneity in the flowing system we may have another in which the nature of the flow, and hence n remains the same, but the numerical value of k varies with the radius. Such a situation was actually encountered in the radial flow experiments as described in Section II where the region near the hole was plugged and hence had a considerably lower permeability than the outer undisturbed sections of the sand sample. If we know how k varies with the radius this problem may be solved by the usual potential theory methods.

A serious effort has been made to get data from several gas companies on the initial stages of decline to test our curves numerically. This has been almost futile because of the inaccuracy of the data and unsteadiness in the field conditions with respect to the back pressure, or even use of the wells. However, the general fact that the orders of magnitude of the decline rates as observed in the field agree with what we have computed is borne out in almost all cases.* It seems, therefore, that in so far as we can test it at present, our theory of gas well decline as based on our experimentally determined laws of flow is able to account for the general features observed in the field. The following points should be added however. In the experimental study of flows for very high pressure gradients and high velocities the pressures must be taken within the sand body, or else corrections should be made for the orifice effect of the sand face. Furthermore, the high productions such as are indicated by Figs. 11 to 14 presuppose that the casing in the well is capable of carrying such productions with a sand face pressure of only 4 atmospheres. In practice this cannot be realized because of the friction in the casing which may amount to several hundred pounds in deep wells. This means that the very high initial productions will automatically be reduced. If necessary our equations can take complete account of this, and only for simplicity, has it been neglected since its inclusion would not appreciably change the essential features of the problem.

V. Concluding Remarks

Although in the present investigation we have not actually reproduced field conditions with regard to the magnitude of the pressures used and the physical dimensions of our flow system, it is felt that our results will suffice to describe the important features of the problem. Probably the whole difference between the gas flow in the underground reservoir and the flow in our laboratory apparatus could be represented by a variation in the values of the exponent n and the permeability constant k. Such variations would also include effects of the nature and composition of the gas and for that reason we have not thought it necessary to study in detail any other gas than air. Our main discussion of our results has treated n and k as general parameters and it would be only a matter of numerical computation to take account of such variations as may be necessary for individual field problems.

The writers wish to thank Mr. R. D. Wyckoff and Dr. Paul D. Foote of this Laboratory for their continued assistance and the many suggestions they have offered during the course of this work.

Note added in proof. Since the present investigation was completed our attention has been called to the work of H. R. Pierce and E. L. Rawlins (U. S. Bureau of Mines Report of Investigations, Serial 2930, May 1929) in which experiments were carried out on actual gas wells. Their results confirm the above work, in particular the predictions of Eq. (11), in that their data in all cases show that the production varies as a power of δp^2, thus justifying the direct application of our analysis to field conditions.

AN EXPERIMENTAL METHOD OF DETERMINING CERTAIN CHARACTERISTICS OF THE FLOW AT THE SURFACE OF AN AIRFOIL

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(RECEIVED APRIL 23, 1931)

ABSTRACT

The heat dissipation per unit area from the surface of model airfoils is experimentally measured by means of narrow resistance strips mounted parallel to the span and flush with the surface. Results, covering variation with distance from the leading edge, with velocity as well as with change of angle of attack, show that the heat dissipation is in no simple way related to the velocity at the strip, as measured by the static pressure, but is evidently characteristic of the airflow at the surface. Further significance of the heat dissipation is sought. The relation of heat dissipation to viscous resistance (Reynold's law) is discussed with relation to (1) two-dimensional flow between parallel plates, (2) a plate plate parallel to the flow and (3) the general case. It is concluded that the law is applicable to (1) and (2) but not to the general case. An integral relation is developed from which it is shown that the heat dissipation, as above measured, is proportional to the velocity gradient at the surface and hence to the viscous resistance. Curves present measurements made upon two airfoils and these are briefly discussed.

AN EXAMINATION of the possibility of preventing ice formation upon airplane wings by use of the waste heat of the engine leads to the problem of the heat dissipation from airfoils. The solution of this problem does not lend itself readily to theoretical treatment for at least two reasons. First, the airflow in the boundary layer of an airfoil, at practical speeds, is probably in part of the turbulent type, and for this kind of motion the exact differential equation to be used is uncertain. Second, even if the equation for the heat flow applicable to laminar airflow be accepted, the solution of this equation with the boundary conditions imposed by the airfoil surface has not, I believe, been accomplished. It is necessary, therefore, to revert to experimental means, and it is desirable to measure not only the heat dissipation from the airfoil surface as a whole, but the energy convected away per unit area as a function of the location of the unit area with respect to the airflow.

The latter has been accomplished by mounting narrow resistance strips parallel to the span of the model airfoil and flush with its surface, and by measuring the rate of loss of energy due to forced convection. The resistance strips chosen were either of nichrome II or chromel A ribbon, and were mounted on mahogany airfoils. Thermocouples led to the under side of the resistance strips and served to measure their temperatures. The heat dissipation, P_c, was measured as the Joule heat, I^2R, where I is the current within the strip and R the resistance of the strip.

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Measurements have been made upon three model airfoils. (See Fig. 1.) 1. The first was a Clark Y-section of 4″ chord and 7″ span with three strips mounted on the back at 0.03, 0.20, and 0.66 x/c (percentage of chord). The resistance ribbon was nichrome II 1/8″×0.007″. 2. The second model was the same section, somewhat modified, and with seven strips mounted on the back of the airfoil at 0.03, 0.11, 0.20, 0.32, 0.49, 0.66, and 0.80 x/c. The resistance ribbon used on this model was chromel A, 1/16″×0.013″. 3. The third model was a symmetrical Joukowski section, modified at the trailing edge, and with nine strips mounted at 0.04, 0.15, 0.25, 0.36, 0.475, 0.575, 0.69, 0.79 and 0.93 x/c. Again, the strips were of chromel A and were 1/16″×0.013″.

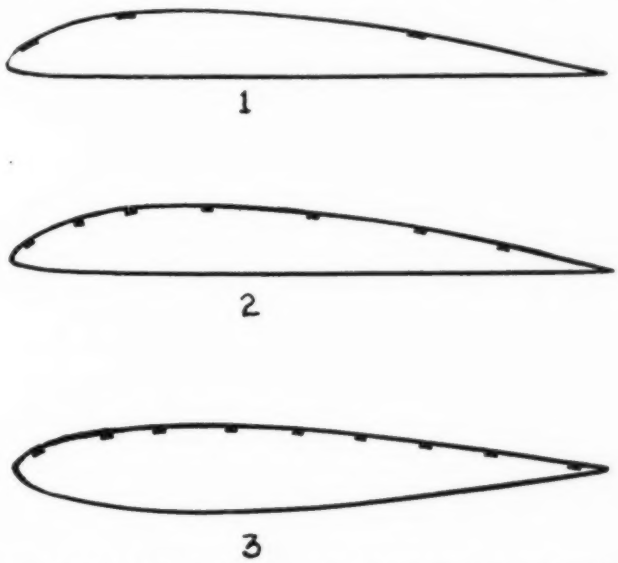

Fig. 1. Three Model Airfoils. Numbers 1 and 2 are Clark Y sections. Number 3 is a modified Joukowski type.

In the case of model 1, arrangement was made for measuring the velocity at the outer limit of the boundary layer, relative to each strip, V_i, by means of the static pressure at the strip. The mean head at the working section was determined from the value of the static pressure in the exit cone and the velocity at the strip was then calculated by the Bernouilli relation. The mean working section velocity was determined by a static tube located in the entrance cone of the tunnel which had been calibrated in terms of a pitot-static in the working section, where the model was to be supported.

In making preliminary observations, it was found that the galvanometer, recording the temperature difference between the strip and the air stream, oscillated erratically. By preventing, as much as possible, air currents into and out of the large room, in which the open-return tunnel was operated, and especially by increasing the length of the entrance to the working section of the tunnel, these oscillations were reduced until any variation due to them was less than the experimental error due, for instance, to the oscillations in

velocity. However, in making the measurements, presented in this paper, there still remained definite oscillations under certain conditions, which by careful correlation could be attributed to instability of the flow. These circumstances were recorded in the data-taking but are not offered here. In all cases, the mean point is recorded, unless the variations are too wide, when either points are omitted and the curves dotted, or two points are plotted, indicating the two principal centers of oscillation.

Measurements upon model 1 showed: (1) the energy dissipated per unit area is proportional to the temperature difference between the strip and the air stream, as shown in Fig. 2. This fact allows of the definition of a heat dissipa-

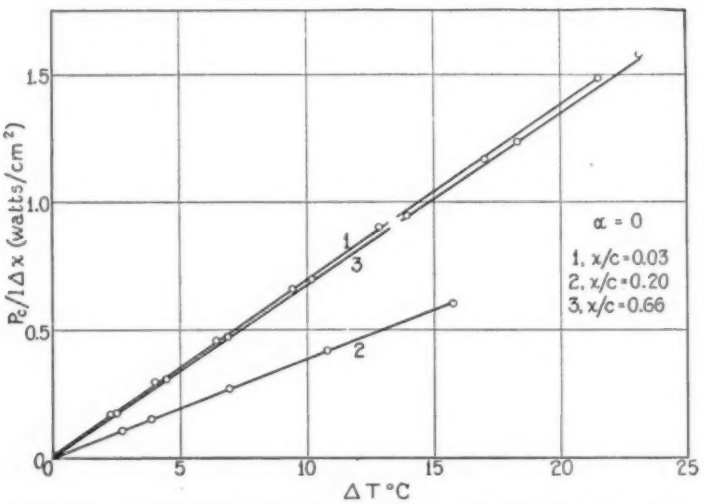

Fig. 2. The variation of the heat dissipation per unit area with temperature difference for the three strips of model 1. Mean working section velocity 19.9 m/sec.

tion coefficient as $p_c = P_c/l\Delta x\Delta T$ where l is the length of strip, Δx the width of strip and ΔT is the temperature difference between the strip and the air stream. The slopes of the curves are dependent upon the location of the strips and hence may be supposed to be functions of the airflow. (2) Fig. 3 indicates the variation of the coefficient, p_c, with the velocity at the strip, V_i. It is to be noted that the curves are again dependent upon the location of the strips with respect to the airflow. Figure 4 shows similar curves for three strips of model 2, save that in this case the mean working section velocity, V, has been plotted and the measurements were made particularly for lower velocities. In curve $a-a$ of this series, certain points were taken by always decreasing the velocity to its value, other points by always increasing the velocity to its value. No consistent difference manifested itself.

When the angle of attack is varied, while the mean working section velocity is held constant,[1] it is found that the heat dissipation, p_c, is in no simple

[1] Due to the large ratio of the cross-section of the wing models to that of the tunnel, the numerical values reported in this article should not be relied upon for values of the angle of attack greater than $+25°$ or less than $-20°$.

way related to the velocity at the strip, but is evidently characteristic of the flow (See Fig. 5). This led to a more careful examination of the significance of the heat carried away by forced convection.

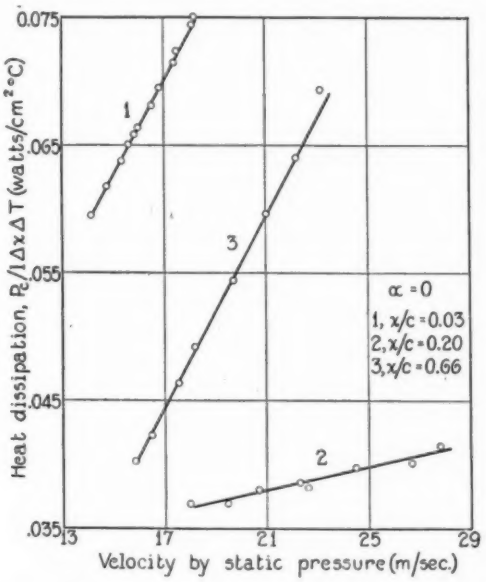

Fig. 3. The variation of the heat dissipation coefficient with velocity for the three strips of model 1.

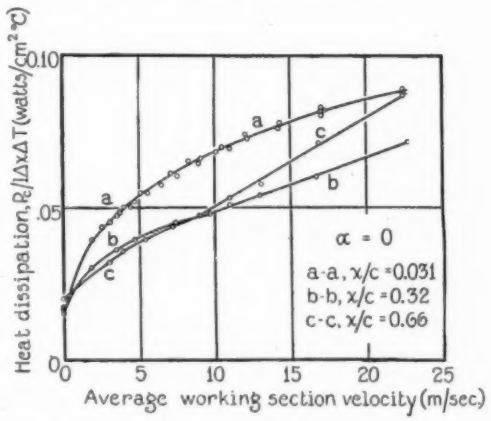

Fig. 4. The variation of the heat dissipation coefficient with mean working section velocity for three strips of model 2.

Analogy of Dissipation of Heat to Viscous Resistance

According to the law of Osborne Reynolds, enunciated by him in 1874, that, "the motion of heat from the surface · · · follows the same laws as the motion of momentum to the surface," the heat dissipation and the viscous resistance should be proportional. A study of this law may be based directly

upon the differential equations and their corresponding boundary conditions for any given problem. These equations for the hydrodynamical and the heat cases are respectively:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{\mu}{\rho}\nabla^2 u - \frac{1}{\rho}\frac{\partial p}{\partial x} \tag{1a}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = \frac{\mu}{\rho}\nabla^2 v - \frac{1}{\rho}\frac{\partial p}{\partial y} \tag{1b}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = \frac{\mu}{\rho}\nabla^2 w - \frac{1}{\rho}\frac{\partial p}{\partial z} \tag{1c}$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z} = \frac{k}{\rho c}\nabla^2\theta + Q \tag{2}$$

where: u, v, w are the components of the fluid velocity; μ is the coefficient of viscosity, and ρ, density of fluid; p, the static pressure within fluid; θ the temperature at a point within fluid; k the thermal conductivity of fluid; c the specific heat of fluid; and Q the source intensity at a point within fluid.

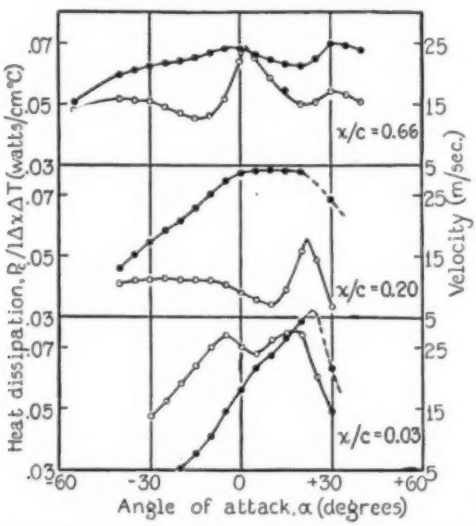

Fig. 5. The variation of the heat dissipation coefficient and of the velocity at the strip, as measured by the static pressure, with angle of attack for the three strips of model 1. Mean working section velocity 19.9 m/sec. ○ coefficient, ● velocity.

In the case of two-dimensional flow past an airfoil, Eqs. (1a) and (2) are those to be observed for this analogy. There is no quantity in heat flow analogous to the pressure in the hydrodynamical consideration, but there are examples, where the pressure gradient may be simulated by an addition of heat, that is, a source proportional to the pressure gradient. On the whole, it may be stated that the analogy is rather fictitious, unless the pressure term vanishes. Consideration of three special problems follows.

Consider the case of two-dimensional flow between infinite parallel, plane plates, (See Fig. 6a) maintained at a temperature different from that of the

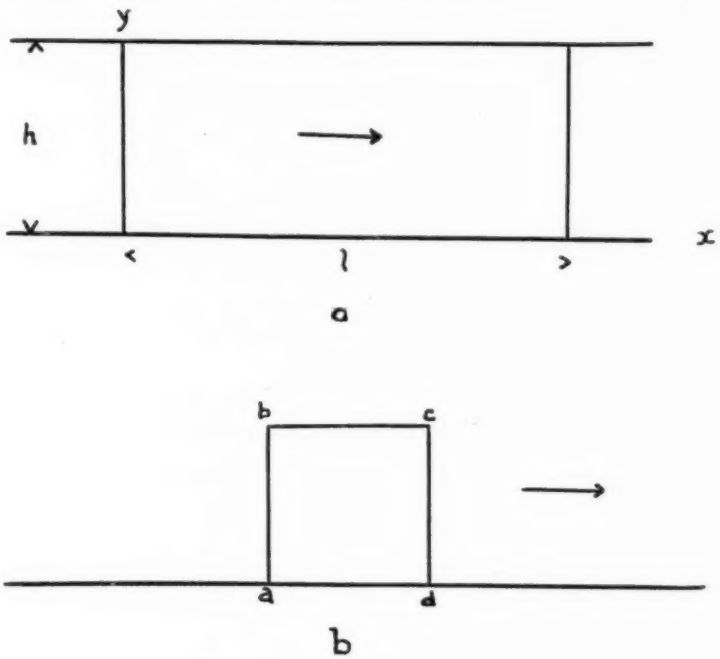

Fig. 6.

fluid. Let the fluid motion be laminar in character so that the velocity is stationary.

$$\partial u/\partial x = 0 \qquad v = 0 \qquad w = 0.$$

Eq. (1) becomes:

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2}. \tag{3}$$

By use of the Bernouilli relation: $\partial p/\partial y = \rho(V^2/R)$, p is seen to be independent of y; hence $\partial p/\partial x = \mu(\partial^2 u/\partial y^2) = c$ where c is a constant. For the corresponding heat problem, Eq. (2) reduces to:

$$u \frac{\partial \theta}{\partial x} = \frac{k}{\rho c}\left[\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right]. \tag{4}$$

The analogy between (3) and (4) is not obvious. However, if two assumptions are introduced, namely that the conduction term, $k(\partial^2\theta/\partial x^2)$, is small compared to the convection term, $\rho c u(\partial\theta/\partial x)$, and that this latter is independent of y, the analogy becomes evident; for, with these limitations:

$$\rho c u \frac{\partial \theta}{\partial x} = k \frac{\partial^2 \theta}{\partial y^2} = c' \tag{5}$$

where c' is a constant. Let the width of the stream be taken as unity and the

distance between the plates as h. Then the proportionality of R, the resistance of a length, l, to P_c, the heat loss for the same length, is to be found in the following expressions by virtue of the above analogy.

$$R = \int_0^l h \frac{\partial p}{\partial x} dx = \int_0^l h\mu \frac{\partial^2 u}{\partial y^2} dx$$

$$P_c = \int_0^l h\rho c u \frac{\partial \theta}{\partial x} dx = \int_0^l hk \frac{\partial^2 \theta}{\partial y^2} dx.$$

A somewhat similar treatment of this problem has been given by Prantl.[2]

If the fluid flow is not laminar, the treatment from a mathematical standpoint is not so obvious. Consideration of the processes of heat dissipation and momentum transfer lead one to the conclusion that the effect of turbulence would be to increase both quantities in exact proportion so that the result still holds. This is borne out by experiment. (See bibliography as given by Rice.[3])

Consider next the case of a flat plate with fluid flow parallel to its plane and again let the flow be of laminar type. In this case $\partial p/\partial x$ vanishes, as may be observed from the two Bernouilli equations:

$$p + \tfrac{1}{2}\rho V^2 = \text{constant} \quad \text{(a)} \quad \frac{\partial p}{\partial y} = \rho \frac{V^2}{R} \quad \text{(b)}. \tag{6}$$

With reference to Fig. 6b, the pressure along ab and cd is constant by 6b. Along bc, the pressure is constant according to 6a, the use of which is allowable by virtue of the Prantl boundary layer theory.[4] Eqs. (1a) and (2) now become:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu}{\rho}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] \tag{7a}$$

$$u\frac{\partial t}{\partial x} + v\frac{\partial t}{\partial y} = \frac{k}{\rho c}\left[\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right]. \tag{7b}$$

The conclusion is that an exact analogy exists between the hydrodynamical and the heat cases and that the rate of heat dissipation and the viscous resistance must be proportional, provided only $\mu = k/c$ as is the case for permanent gases.

Similar considerations, as mentioned above, lead one to believe that the conclusion would not be altered for a turbulent fluid stream.[5]

In the general case, the equations are 1a and 2. There is no analogy between the two equations and no reason to expect that the rate of heat dis-

[2] L. Prantl, Phys. Zeits. No. 11, p. 1072 (1910).

[3] C. W. Rice, Industrial and Engineering Chemistry, page 460, May 1924.

[4] Handbuch der Physik VII, Geiger and Scheel, p. 121.

[5] Since preparing this paper, there has come to my attention the following rather complete discussion of this problem, together with references: N.A.C.A. Tech. Mem. No. 614, which consists of a translation of an article, The Transference of Heat from a Hot Plate to an Air Stream, by Franz Elias, Abh. a.d. Aero. Inst. a.d. Tech. Hoch. A., No. 9, 1930.

sipation and the viscous resistance would bear any simple relation to one another. In the case of the narrow resistance strip mounted flush with the surface of a non-conducting airfoil and maintained at a temperature different from that of the air stream, it might be thought that the analogy of the equations would hold, since for a narrow strip $\partial p/\partial x$ might be regarded as negligible. However, the admission of the analogy is not clear, for the boundary conditions corresponding to the two equations are different, namely $v=0$ for the entire airfoil surface, whereas $\theta=\theta_s$ only for the short width of the strip.

An Integral Relation

An integral relation has been developed for the thermal boundary layer as follows. With reference to Fig. 7, the airfoil surface is indicated by *ss*, drawn to so large a scale as to make it appear flat. AB is the resistance strip, maintained at temperature θ_s. The fluid is maintained at temperature θ_0, such that $\theta_s-\theta_0=\Delta T$, and its flow is indicated by the arrow. Curvilinear coordinates are chosen as indicated in the figure.

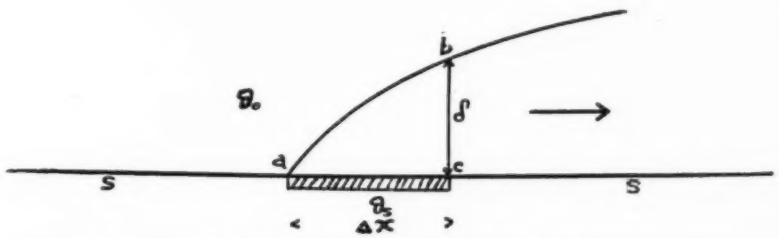

Fig. 7. Schematic sketch of thermal boundary layer relative to a strip.

A thermal boundary layer may next be defined relative to the strip as included by the value of y at which $\partial\theta/\partial y=\epsilon$, a small quantity. It follows that the temperature of the fluid at the boundary line, ab, is as near θ_0 as may be desired. The momentum crossing the line bc per unit time per unit depth is

$$\int_0^\delta \rho u dy.$$

This also, by the continuity equation, $\partial u/\partial x+\partial v/\partial y=0$, is the corresponding rate at which momentum is transported across the boundary line ab. For the volume included by the area abc of depth unity, the following equation holds by virtue of the conservation of thermal energy:

$$-\theta_0 c\int_0^\delta \rho u dy+\int_0^\delta \rho c u\theta dy+\int_0^\delta k\left[\left.\frac{\partial\theta}{\partial x}\right|_2-\left.\frac{\partial\theta}{\partial x}\right|_1\right]dy$$
$$+\int_0^{\Delta x} k\left.\frac{\partial\theta}{\partial y}\right|_{ab}-\int_0^{\Delta x} k\left.\frac{\partial\theta}{\partial y}\right|dx=0 \quad (8)$$

wherein, besides those symbols previously defined,

$\left.\frac{\partial\theta}{\partial x}\right|_1$ = temperature slope at boundary line ab

$\left.\frac{\partial\theta}{\partial x}\right|_2$ = " " " " " ac

$\left.\frac{\partial\theta}{\partial y}\right|_{ab}$ = " " " " " ab

$\left.\frac{\partial\theta}{\partial y}\right|_0$ = " " " surface of strip.

Now:

$$\rho c \int_0^\delta (\theta - \theta_0) u\,dy + \int_0^\delta k \left[\left.\frac{\partial\theta}{\partial x}\right|_2 - \left.\frac{\partial\theta}{\partial x}\right|_1\right] dy - \int_0^{\Delta x} k \left.\frac{\partial\theta}{\partial y}\right|_{ab} dx - \int_0^{\Delta x} k \left.\frac{\partial\theta}{\partial y}\right|_0 = 0 \quad (9)$$

The second or conduction term of (9) may be neglected in comparison with the first or convection term. The third term may be made as small as is desired and the last term is the rate of heat dissipation from the strip, P_c. Eq. (9) becomes

$$\rho c \int_0^\delta (\theta - \theta_0)\, u\,dy = k \int_0^{\Delta x} \partial\theta/\partial y\,|_0 dx = P_c \quad (10)$$

By use of the average value theorem:

$$\rho c \bar{u} \int_0^\delta (\theta - \theta_0) dy = k \frac{\overline{\partial\theta}}{\partial y} \Delta x = P_c \quad (11)$$

where $\overline{\partial\theta}/\partial y$ is the average value over Δx.

In the case of laminar flow and turbulent flow as well, the second derivative of the velocity, normal to the surface, $\partial^2 u/\partial y^2$, practically vanishes at the surface.[6] Therefore the average value of u over the boundary bc may be calculated as:

$$\bar{u} = \frac{1}{\delta} \int_0^\delta \frac{\overline{\partial u}}{\partial y} y dy = \frac{\delta}{2} \frac{\overline{\partial u}}{\partial y} \quad (12)$$

where $\overline{\partial u}/\partial y$ is an average value over bc. If the temperature of the strip, θ_s, is indefinitely reduced, the thermal boundary layer thickness, δ, is correspondingly decreased and must certainly lie within the narrow region close to the surface, in which the higher derivative of the velocity vanishes. Experimentally, the rate of heat dissipation, P_c, was found to be proportional to the temperature difference between the strips and air stream, ΔT. So long as this relation holds, the values $\bar{u}$ and $\overline{\partial u}/\partial y$ may be interpreted as values within this thin layer and, in fact, $\overline{\partial u}/\partial y$ may be regarded as $\partial u/\partial y\,|_0$, the velocity gradient at the surface.

[6] Handbuch der Physik VII, Geiger and Scheel, p. 144.

From the above considerations, Eqs. (11) and (12), the following is obtained:

$$\left[\frac{\delta}{2}\int_0^{\delta}\rho c(\theta-\theta_0)dy\right]\frac{\partial u}{\partial y}\bigg|_0 = k\frac{\overline{\partial\theta}}{\partial y}\Delta x = P_c. \tag{13}$$

Once more using the average value theorem, (13) becomes:

$$\left[\frac{\delta^2}{2}\rho c\frac{\overline{\theta-\theta_0}}{\Delta x}\right]\frac{\partial u}{\partial y}\bigg|_0 = k\frac{\overline{\partial\theta}}{\partial y} = p_c \tag{14}$$

where $\overline{\theta-\theta_0}$ is the average value of this difference over the boundary bc and p_c is the average heat dissipation per unit area of the trip.

It appears that the analogy of the hydrodynamical to the thermal problem would obtain, provided the bracketed expression of Eq. (14) were shown to be a constant, for in this event:

$$K\left[r = \mu\frac{\partial u}{\partial y}\bigg|_0\right] = \left[k\frac{\overline{\partial\theta}}{\partial y} = p_c\right] \tag{15}$$

where r would be the viscous resistance per unit area. The bracketed expression[7] is constant, save in the immediate neighborhood of the stagnant points, as may be proved in the following manner. For a given set of steady conditions:

$$\delta^2\frac{\partial u}{\partial y}\bigg|_0 = \frac{p_c\Delta x}{\rho c\overline{\theta-\theta_0}} = \text{constant} \tag{16}$$

and if the set of experimental values is chosen as:

$$p_c = \frac{0.5}{4.18}\ \text{cal/cm}^2$$

$$\Delta x = 0.317\ \text{cm}$$

$$\rho = 0.001196\ \text{gm/cm}^3$$

$$c = 0.237$$

$$\overline{\theta-\theta_0} = \frac{\theta_s-\theta_0}{2} = 3$$

the constant of (16) equals 89. Write b for $\partial u/\partial y\,|_0$ and differentiate with respect to b.

$$b\frac{\partial(\delta^2)}{\partial b} + \delta^2 = 0$$

$$\frac{\partial(\delta^2)}{\partial b} = -\frac{\delta^2}{b} = -\frac{89}{b^2}.$$

[7] By plausible assumptions, it is possible to compute the bracketed expression by use of experimental data. For instance: assume $\overline{\theta-\theta_0}$ independent of Δx and $\delta=\Delta x^{\alpha}$, then by use of strips, similarly located with respect to the airflow, but of different widths:

$$\left(\frac{\Delta x_1}{\Delta x_2}\right)^{2\alpha-1} = \frac{p_{c_1}}{p_{c_2}}$$

from which α is known. The expression may then be estimated by choosing a value for $\overline{\theta-\theta_0}$.

For variations in δ^2 to be equal or less than one per cent of those in b

$$\frac{\partial(\delta^2)}{\partial b} = -\frac{89}{b^2} \leqq 0.01$$

$$b^2 \geqq 8900$$

$$b \geqq 94 \frac{1}{\text{sec}}.$$

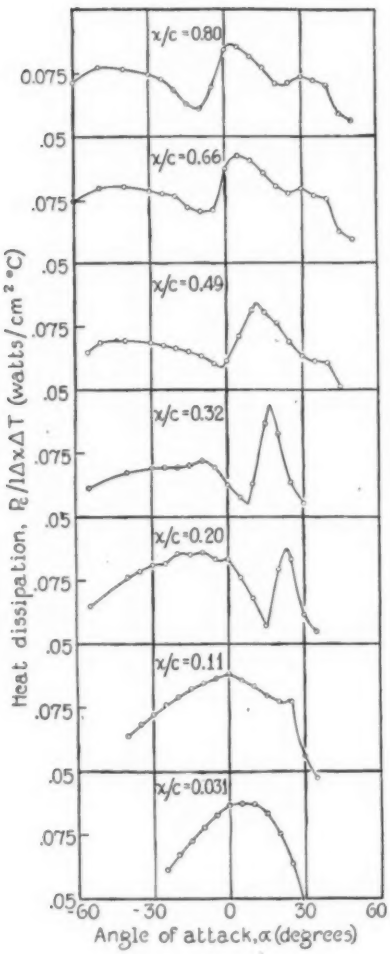

Fig. 8.

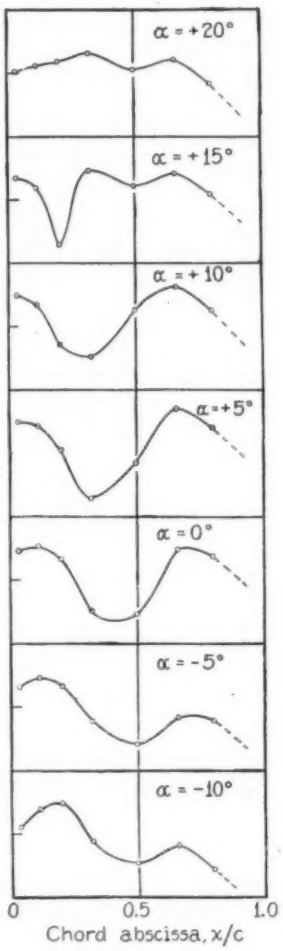

Fig. 9.

Fig. 8. The variation of the heat dissipation coefficient with angle of attack for the strips of model 2. Mean working section velocity 19.9 m/sec.

Fig. 9. The variation of the heat dissipation coefficient with percentage chord abscissa for the strips of model 2, angle of attack as parameter. Data taken from Fig. 8.

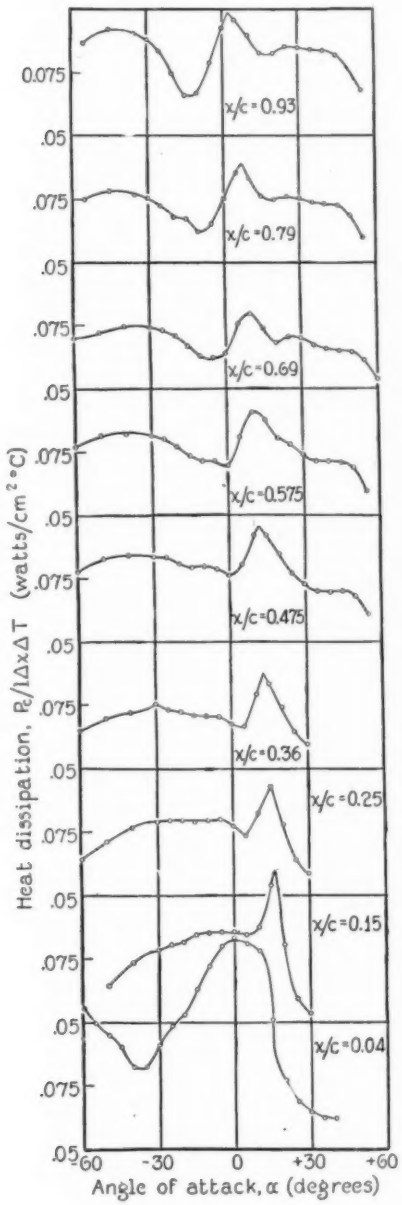

Fig. 10.

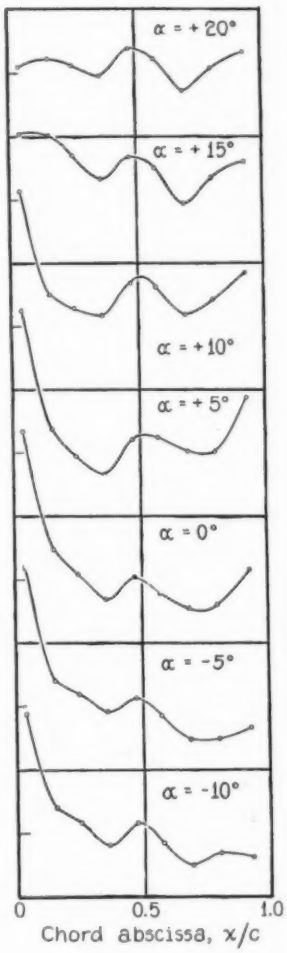

Fig. 11.

Fig. 10. The variation of the heat dissipation coefficient with angle of attack for the strips of model 3. Mean working section velocity 19.9 m/sec.

Fig. 11. The variation of the heat dissipation coefficient with percentage chord abscissa for the strips of model 3, angle of attack as parameter. Data taken from Fig. 10.

From data given by Fage and Falkner[8] on the velocity gradient at the surface of an airfoil, a typical value is chosen, using their notation as:

$$\frac{f}{\rho V_0^2} = \frac{\mu \frac{V}{z}}{\rho V_0^2} = 2.75 \cdot 10^3$$

$$\frac{V}{z} = \frac{2.75 \cdot 10^3 \cdot 0.00237 \cdot 60^2}{3.82 \cdot 10^{-7}} = 6.15 \cdot 10^4 \frac{1}{\text{sec}} .$$

That is:

$$b \doteqdot 6\ 10^4 \frac{1}{\text{sec}} .$$

Hence it appears that for moderate velocities b is always greater than necessary to make changes in δ^2 negligible. It is proved, therefore, that save in the immediate neighborhood of the stagnant points, the rate of heat dissipation per unit area from a strip is proportional to the velocity gradient at the surface and hence to the viscous drag.

The measurements of the heat dissipation coefficients upon the second and third airfoils are shown plotted in Figs. 8, 9, 10 and 11. In all cases, the mean working section velocity was held constant at 19.9 *m/sec*. No attempt is made in this paper to study these curves. However, one outstanding point may be mentioned. In the case of the modified Clark *Y* airfoil, as observed in Figs. 8 and 9, an instability, recognized by a low value of the dissipation coefficient, followed by a considerably higher one, develops at rather small or even negative angles of attack; and with increasing angle moves forward. This instability does not reach the leading edge, for when it arrives at about 0.10 x/c, the condition of burble has fully developed and the flow appears to be considerably changed. This same observation applies to the symmetrical airfoil, as may be seen from the curves of Fig. 10, but the effect is not so obvious in Fig. 11, where the abscissa is x/c. In conclusion, it should be remarked that there appear to be many useful studies in connection with the airflow problem that might be made by use of this method.

The writer acknowledges his appreciation of the interest, helpful suggestions and criticism of Professor C. C. Murdock.

[8] Fage and Falkner, Proc. Roy. Soc. **A129,** 378 (1930).

THE DEPOSITION OF DUST ON WALLS[1]

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(Received May 6, 1931)

Abstract

An apparent uncertainty exists in literature dealing with the cause for the peculiar deposition of dust on plaster and lath walls wherein the course of the laths and rafters behind the plaster is outlined in dust, popularly called "lath marks." Conclusive experimental proof of a thermal cause is given which is in agreement with the general theory of the behavior of small particles suspended in an atmosphere in which a temperature gradient exists. Experimental evidence is produced to show that this phenomenon may be reduced to a negligible degree by thermal insulation.

Introduction

A CIRCULAR of the Bureau of Standards[2] entitled "Wall Plaster, Its Ingredients, Preparation, and Properties" states "Sometimes discoloration, due to collection of dust, outlines the units of the backing to which plaster is applied. A possible explanation for this is as follows: Plaster is usually cooler than the air in contact with it. Frequently it is so much cooler that it is below the dew point, and moisture is condensed out of the air and deposited on the plaster. This moisture is drawn by suction from the surface into the interior of the plaster. Eventually the plaster becomes saturated and some moisture remains on the surface. This collects dust from the air. Regarding the plaster as a reservoir for the absorption of water, it will be seen that the space between the backing units will absorb more water than the unit. Therefore the space in front of the unit will become saturated more quickly, and then discolored. If the discoloration occurs over the joints instead of over the units, it means simply that the unit is more absorptive than the plaster. This type of defect is most noticeable with wood lath and clay tile. It does not occur with metal lath. It can be remedied by painting the plaster."

In connection with the phenomenon of dust which outlines the course of the laths and studding behind the plaster and also often indicates the position of nails holding lath or wall board to studding, it is worthy of note to recall the work of Tyndall[3], Rayleigh[4], Aitkin[5], Lodge[6], and Clark[7], done between 1870

[1] An abstract of this paper appears in the program of the November 28–29, 1930 meeting of the American Physical Society.

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[2] Circular of the Bureau of Standards, No. 151, 1924, p. 58.

[3] Tyndall, Proc. Roy. Inst. London **6,** 1 (1870).

[4] Rayleigh, Proc. Roy. Soc. Dec. (1882); also Nature XXVIII, p. 139.

[5] Aitken, On the Formation of Small, Clear Spaces in Dusty Air. Trans. Roy. Soc., Edinburgh **32,** 239–272 (1884). Also Collected Scientific Papers (Cambridge 1923).

[6] Oliver J. Lodge, On Lord Rayleigh's Dark Plane, Nature **28,** 297–299 (1883), O. J. Lodge, and J. W. Clark, On the Phenomena exhibited by Dusty Air in the Neighborhood of

and 1885. These men did considerable experimental work with dust to explain the existence of the region of warm, dust-free air, immediately surrounding and streaming upwards from a hot rod as illustrated in Fig. 1. It was observed that a cold rod caused a down-streaming current of dust-free air and in this case dust was deposited upon the rod instead of being repelled. On the basis of this work Poynting and Thomson[8] in their textbook entitled "Heat", and Gibbs[9] in his book entitled "Clouds and Smokes" attribute the peculiar deposition of dust on lath and plaster walls to a thermal cause. On page 152 of "Heat" we read, "Where the plaster is backed by wood, it is probably kept warmer, and the dust is not so freely deposited as on the neighbouring cooler parts."

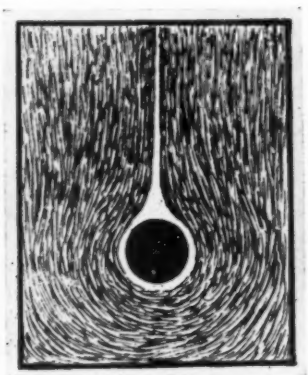

Fig. 1. An illustration of the dust-free space about a hot rod in a dusty atmosphere.

The writer has discovered that the lay public and especially artisans working with plaster, wall decorations, or woodwork hold many quite decided and divergent views regarding the cause of "lath marks." Most popular, perhaps, of the beliefs held by the lay public regarding this cause is that the plaster is porous to the extent of producing a sort of filter-like action on currents of air passing from the room to the inner wall behind the plaster. Electrical, radioactive, gravitation, and radiation influences have also been suggested as possible causes.

Experimental Procedure

In view of the uncertainty of the cause of this phenomenon experimental evidence seemed highly desirable and an exhaustive investigation was made. Observations in many dwellings revealed that a treatment of the wall surface by waxing, painting, enameling, and ordinary wall papering does not appear to affect the peculiar affinity of dust for certain regions on the wall, particu-

Strongly Illuminated Bodies, Proc. Phy. Soc. London **6,** 1–29 (1884). Also Phil. Mag. **17,** 214–239 (1884). O. J. Lodge, Nature **29,** 610–613 (1884). Nature **31,** 265–269 (1885). See also Presidential Address, Jour. Roy. Sanitary Inst. **27,** 42 (1906).

[7] Clark, ref. 6, O. J. Lodge, and J. W. Clark,

[8] Poynting and Thomson, Text-book of Physics "Heat" (1925), C. Griffin and Co. Ltd.

[9] Wm. E. Gibbs, Clouds and Smokes, (1924), Blakiston's and Sons.

larly those regions over the spaces between the lath. An exaimination of the phenomenon reveals that the dust can be most readily and completely wiped off the wall, and demonstrates positively that there is no penetration of the dust into the wall.

In order that the phenomenon might be studied at first hand a portable experimental wall section approximately 2×6 feet was built up of lath and plaster on two pieces of studding. The plaster on this wall section was sized and papered in the ordinary manner. This wall section was laid upon a laboratory table. A rectangular sheet metal cover was made large enough to cover the papered section of the wall and in addition an ordinary kerosene lamp was placed at one end of the section. By turning the wick of the burning keresene

Fig. 2. A wall section on which "lath marks" were obtained when cold air was drawn through the inner wall space (connected with a chimney).

lamp up a little, this cover afforded a smoke box which produced a deposition of soot on the wall paper. Depositions of soot obtained in this manner were perfectly uniform, with no evidence of striation in "lath marks". The soot was therefore carefully cleaned off the wall paper and another experiment tried. This time a cold draft of air was drawn through the inner wall space between the plaster and the table top during the process of the deposition of soot, by connecting one end of this space with a chimney in the building having a good draft and permitting cold air to enter the laboratory room through open windows. Upon examination of the soot deposit obtained in this manner a splendid reproduction of the "lath marks" was found, a photograph of which is shown in Fig. 2.

It was discovered that if air was blown over a piece of plaster it became negatively electrified. It is well known that dust is attracted to electrified bodies. In order, therefore, to eliminate the possibility of any electrical influence being present during the formation of these "lath marks," and also positively to eliminate the possibility of convection currents passing through the plaster, as well as moisture coming through from the plaster, a wall section was next built up which consisted of wood lath and plaster in which was em-

bedded a large 24 × 36 inch sheet of window glass 0.088 in. thick, the underside of which was covered with aluminum foil 0.0019 in. thick which was attached to the glass by means of heavily waxed paper. The upper surface of the glass formed the outside of the wallsection and was papered in the usual manner. A cold draft of air was now circulated in the inner wall space, this time a small quantity of solid carbon dioxide being used to aid in reducing the temperature beneath the wall. After operating the smoke chamber on this wall section made up of wood, plaster, metal, wax, glass, and paper, a deposition of soot was obtained on the surface with the "lath marks" very beautifully in evidence as shown by the photograph in Fig. 3. There was a design on the wall paper used in this experiment which should not be confused with the dust striations which reveal the regular spacing of the lath. This experiment def-

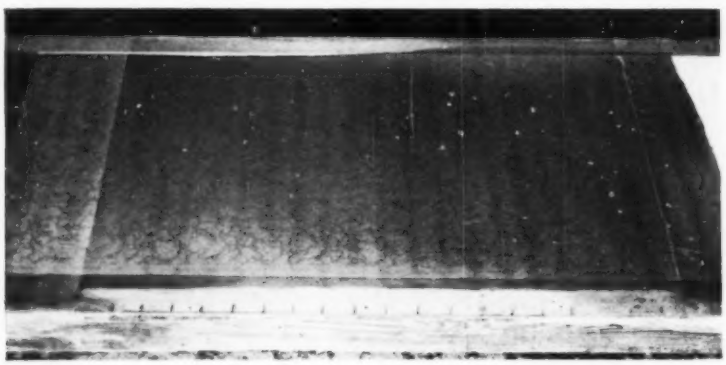

Fig. 3. "Lath marks" obtained on a wall made up of wood lath, plaster, metal foil, wax, window glass and paper. The design on the wall paper should not be confused with the dust striations which reveal the regular spacing of the lath.

initely rules out moisture from the plaster, porosity of the plaster, and electrical charges on the back of the plaster, as possible causes of the lath marks. Radium A deposited on the back of a wall section as well as an intense beam of γ rays passing through the plaster were found not to influence the deposition of dust in the slightest observable way. Even when wires raised to a potential of several thousand volts were stretched beneath a wall section made up of wall board and plaster no striation of dust could be obtained on the wall paper above the wires, due to the fact that the wall was sufficiently conducting to screen out the electric field.

It has been shown by Aitken[10] that a warm, moist surface from which water is evaporating repels dust more effectively than a warm dry surface. This action is especially illustrated when dust is inhaled. It is largely prevented from being deposited on the lungs by the evaporation of water vapor and the repelling action of the warm surfaces. A small area of wall paper was dampened with water at room temperature prior to being exposed to the deposition of soot. This area was practically spotless after a continued period

[10] Aitken, reference 5.

of "smoking" had taken place to the extent that the surrounding paper was black with the deposition of soot. This result is at variance to the views previously set forth in the circular of the Bureau of Standards.[11]

A very interesting experiment was next performed. By protecting half of the papered surface of the wall section with a covering a deposit of soot showing the lath marks was obtained on the exposed half, during which a cold draft of air was circulating beneath the lath and plaster. The "lath marks" obtained in this manner were now carefully covered and thus protected while the clean half was exposed to the deposition of soot, this time, a warm draft of air being circulated beneath the plaster by means of a heating coil and fan. The deposit of soot obtained on this half of the wall section showed a complete reversal of form, the soot striations now appearing over the lath instead

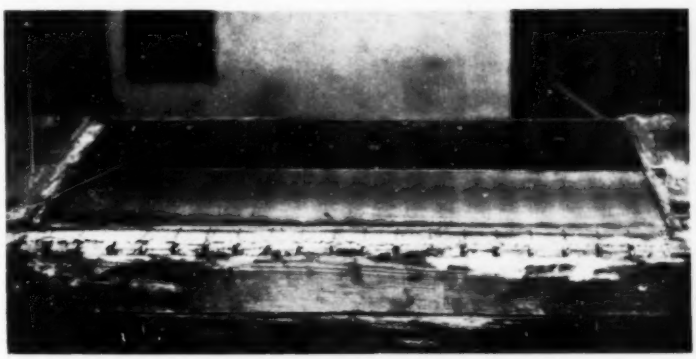

Fig. 4. "Lath marks" showing the reversal effect obtained when the inner wall space is first cooled during the deposition of soot and then heated.

of between the lath as previously obtained. Fig. 4 is a photograph of the deposition of soot thus obtained and shows clearly the reversal of the" lath marks."

Because of the disfiguring the unsightly nature of "lath marks" on walls it seemed desirable to investigate the possibility of reducing or eliminating this phenomenon by means of thermal insulation in the inner wall space. (The writer's attention has been especially directed to the lack of "still air" insulation in the inner wall space of the ordinary frame dwelling. Convection currents of considerable magnitude have been found coming from small openings in walls which to all appearances seemed otherswise tight.) For this investigation a wall section was built up to represent the complete outside wall of the ordinary frame dwelling. It consisted of standard size clapboards, building paper, plain sheathing boards 7/8″ thick, an air space formed by the 2×4 studding, wood lath and plaster, the latter being papered in the usual manner. The inner wall space of this wall section was equally partitioned, or divided into two spaces. Into one of these spaces a modern insulating medium, called "balsam-wool," consisting of light fluffy wood fibre held between two

[11] See reference 2.

sheets of paper, was installed. The other space was left emtpy. A chamber was built around the entire clapboard side of this wall section and the air therein next to the wall maintained at approximately zero degrees Fahrenheit by means of solid carbon dioxide which was scattered over the bottom of the chamber. The smoke chamber was operated over the papered wall surface and the atmosphere therein maintained at approximately eighty degrees Fahrenheit. Fig. 5 is a photograph of the soot deposit thus obtained. It is interesting to note the damping action of the balsam-wool on the flow of heat through the wall as indicated by the very marked reduction in the intensity of the "lath marks" on the insulated side. It is very evident from this photograph that "lath marks" are good indicators of the regions on a wall where

Fig. 5. Reducing the intensity of "lath marks" by thermal insulation in the inner wall.

heat is most rapidly passing through it. To further confirm this point thermocouples were embedded in the plastered surface and the temperature of the wall at various points was measured under conditions suitable for the production of "lath marks." A radiation thermopile supported close to the surface of the wall was also used for this purpose and found to be more suitable in making relative determinations of temperature. It was found that the regions backed by wood lath were warmer than those between the lath. Likewise it was also found that regions directly over nail heads on the studding were colder than neighboring regions. The differences in temperature of the various regions of the wall surface whose temperatures were thus measured appeared in general to run about half a degree Centigrade although these differences were necessarily variable. These results are to be expected from a consideration of the thermal conductivities of the materials involved. It is of interest to note that mortar plaster is listed[12] as having a thermal conductivity about

[12] Coefficient of thermal conductivity of mortar plaster is 0.0013 (gm cal. per cm^3 per degree C per sec.) whereas that of fir wood perpendicular to the axis of the grain is 0.00009 (Hudson, The Engineer's Manual, John Wiley and Son.—Handbook of Chemistry and Physics, Chemical Rubber Co.).

fourteen times greater than that of fir wood perpendicular to the grain. Different kinds of plaster will of course have different conductivities but gypsum base plasters should in general have thermal conductivities much greater than that of wood perpendicular to the grain.

In the foregoing experiments the surface of the experimental wall section was maintained at a temperature considerably above the dew point within the smoke chamber. For this reason it is clearly evident that the condensation of water vapor on the wall is not the agency which produces "lath marks."

Conclusions

The mechanism whereby dust is caused to settle in "lath marks" appears to be identical to that causing the dust-free region about a hot body, (Fig. 1.) studied by Lodge[13] and others.[14] Regarding this mechanism Lodge[15] writes "The dust is kept away by molecular bombardment. It has been shown by Tait and Dewar, and by Osborne Reynolds, that a Crookes bombardment is effective at even ordinary pressures provided the bodies bombarded are small. Dust particles are very small, and so they get driven by molecular impact away from hot surfaces and toward cold ones." He further states[16] "It is difficult to give the full explanation of the dust free spaces in a few words, but we may say roughly that there is a molecular bombardment from all warm surfaces by means of which small suspended bodies get driven outwards and kept away from the surface. It is a sort of differential bombardment of the gas molecules on the two faces of a dust particle somewhat analogous to the action of Mr. Crookes' radiometer vanes. Near cold surfaces the bombardment is very feeble and if they are cold enough it appears to act towards the body, driving the dust inward—at any rate there is no outward bombardment sufficient to keep the dust away, and bodies colder than the atmosphere surrounding them soon get dusty." Albert Einstein[17] has shown that a small body suspended in an atmosphere in which a temperature gradient exists should experience a resultant force due to molecular bombardment repelling it away from the warmer region.

The foregoing experiments thus support the view of Sir Oliver Lodge[15] that, "Whenever the air is warmer than bodies it deposits its dust and smoke upon them; whenever bodies are warmer than the air they keep the dust off, except when the weight of some of the larger particles is sufficient to overcome the bombardment; a thing which is very likely to happen on a horizontal and slightly warm surface."

In the production of "lath marks" as ordinarily found on the walls of dwellings the plaster which is backed by wood lath is more protected and thus

[13] Lodge, reference 6.

[14] See references 3, 4, 5, 6 and 7.

[15] O. J. Lodge, Nature **31,** 268–269 (1885).

[16] O. J. Lodge, Nature **29,** 612 (1884).

[17] A. Einstein, Zeits. f. Physik **27,** 1 (1924). See also L. Loeb, Kinetic Theory of Gases (1927), McGraw Hill Book Co., pp. 303–305.

thus kept warmer than the plaster between the lath and thus repels dust more effectively. When the inner wall space is heated the plaster between the lath conducts the heat to the outer wall more readily than does the wood lath and hence the regions between the lath repel the dust most effectively, in this case causing the reversal effect illustrated in Fig. 4.

Inasmuch as "lath marks" could be produced when, and only when, the inner wall space was either heated or cooled, and since the reversal effect indicates clearly the part played by the plaster and wood lath in conducting heat to and from the surface of the wall, giving measurable temperature differences to the wall surface, a thermal cause for this phenomenon appears to be unmistakably demonstrated.

The problem of reducing or eliminating "lath marks" on wood lath and plaster walls[18] which is of interest to the general public, thus reduces itself to one of providing adequate thermal insulation in the inner wall sufficient to insure a practically uniform temperature over the surface of the plaster. The marked reduction in the intensity of the "lath marks" when balsam-wool was installed in the wall section (See Fig. 5) is a demonstration of the practicability and possibility of accomplishing this, as well as affording visual evidence of the relative effectiveness of such insulation.

Several problems have suggested themselves during the course of these experiments which will be given further experimental investigation and more information on this subject should be forthcoming within the near future.

The writer wishes to express his appreciation to The Wood Conversion Company of Cloquet, Minnesota, for their interest in supporting this work and to Mr. Howard B. Dailey and Kenneth Warren of this laboratory for their assistance in the experimental work.

[18] The writer has just received a copy of Building Research Bulletin No. 10 entitled "The Prevention of Pattern Staining of Plasters," by D. G. R. Bonnell and L. W. Burridge, published by the Department of Scientific and Industrial Research, England, February 1931. The explanation given in this paper for the cause of "lath marks" is in agreement with the conclusions of the writer (previously announced Phys. Rev. Jan. 1, 1931, p. 111) although a critical analysis of possible causes is not undertaken. An excellent treatment of possible ways of producing an approximately isothermal wall surface is given.

THE EDITOR'S COLUMN

THIS issue of PHYSICS marks the launching of a new journal of the American Physical Society. I hope it may slip quietly into the waters and prove to be a sturdy and useful vessel, worthy of the other two members of the fleet.

On an occasion of this sort it is perhaps appropriate to mention the conditions which made the establishment of a new journal seem desirable. The PHYSICAL REVIEW has hitherto been the only outlet provided by the American Physical Society for the publication of original research. As you may have noted the Review has more than doubled in volume during the past six years and has become more and more the exponent of the purely introspective side of physics. This is but the natural result of the fundamental and radical changes in the logical framework of the science which have attracted the attention of an ever increasing group of active physicists. But fascinating as are the developments in atomic physics and the quantum mechanics, they do not by any means represent the whole of the science, nor are they more interesting or valuable than the original work of the greater number of professional physicists who are applying physical methods and principles to the problems of other sciences and the industries. Because, however, of the conjested state of the Physical Review and of its specialized character the work of this latter group of physicists has been scattered through a number of engineering, chemical and industrial journals. Such scattering is obviously unfortunate and we hope that the new journal, PHYSICS, will become a medium for the stimulating exchange of the discoveries, methods and ideas of this large group of research men.

In addition to articles of the type suggested above we are planning other features; in the first place the readers of PHYSICS will be interested in following the recent advances in the Engineering sciences and will be glad to learn that beginning with the second issue there will be a section devoted to Engineering News and Notes.

A second feature will be found in the Editor's Column. It has been suggested that while the majority of the readers of PHYSICS will not have the time or the specialized knowledge to digest the articles appearing in the PHYSICAL REVIEW, most of them will be keenly interested in learning what the quantum mechanists are accomplishing and what they are attempting to do. For this reason there will be each month in the Editor's Column a brief summary of recent articles along these lines. We are very fortunate in being able to present to our readers the following summary which Professor J. H. Van Vleck of the University of Wisconsin has made of a number of articles appearing contemporaneously in the Physical Review.

Valence Forces in Chemistry

ONE of the great achievements of quantum mechanics has been to make intelligible the subject of valence in chemistry. The two keys to the situation are the Pauli exclusion principle and the Heisenberg exchange effect. The Pauli exclusion principle, which Professor Mendenhall characterizes as "social legislation to prevent over-crowding of electrons," decrees that the K shell admit only 2 members, the L shell only 8, etc. thereby giving a theoretical basis for the groups of 2, 8, 8, 18, 18, 32 in the periodic table.

The Heisenberg exchange (Austausch) effect is rather difficult to describe in popular, i.e. classical language, for the simple reason that it has no real analogue in classical mechanics. It arises from a peculiar quantum-mechanical term which is due to the possibility of the electrons swapping places. Superficially it has the same effect as introducing a potential energy between electrons which is very sensitive to the relative alignment of their spins. Dirac has shown that this potential can be regarded as having the form

$$V_{ex} = -\tfrac{1}{2}J(1+4\mathbf{s}_1\cdot\mathbf{s}_2), \qquad (1)$$

where J is a certain "exchange integral," dependent on the amount the wave functions overlap when permuted, and $\mathbf{s}_1\cdot\mathbf{s}_2$ is the scalar product $|s_1||s_2|\cos(s_1, s_2)$ of the spin angular momentum vectors of the two electrons be-

tween which the exchange ties are being considered.

In the Heitler-London theory of valency it is assumed that commonly the factor J in Eq. (1) is negative. If this assumption $-J>0$ is correct, Eq. (1) shows that the configuration will be one of minimum energy when the scalar product $s_1 \cdot s_2$ has its minimum value. As this product is proportional to cos (s_1, s_2), such a minimum will be achieved when the spins of the two electrons are anti-parallel. With anti-parallel alignment, the vector sum of the two spins is zero, and when this is the case we shall say that two electrons are paired. One now sees why the Lewis concept of paired electrons has been so useful in chemistry. A chemical bond may be regarded as an agreement of two electrons to be as lazy as possible—i.e. seek minimum energy, and this they can do by having their spins anti-parallel. Two paired electrons, with their anti-parallel spins, may be said to represent a saturated valence. On the other hand an electron whose spin is not yet paired with some other one then clearly has an unsaturated valence.

The reader should guard against the impression that the valence forces associated with (1) are due to magnetic fields between spins, even though it is true that the spins make each electron a tiny magnet. Magnetic forces between atoms are negligible, and the semblance (1) of coupling between spins is because the Pauli exclusion principle correlates each type of symmetry in the orbital wave functions with a certain particular alignment of spin. The electrons are interested primarily in low orbital exchange energy, and when they achieve this, Pauli's rules demand that the spins be aligned in a particular way (usually antiparallel). Thus the spin alignments, though unimportant in themselves, act as sign-posts for the type of orbital arrangement, and it is for this reason that the spin appears in (1) even though (1) represents purely orbital energy.

As a matter of fact, precious little is known accurately about the sign of J in (1). That J is really negative has been verified for a pair of hydrogen atoms in their lowest state, and the Heitler-London valence theory may be regarded as a bold extrapolation of this result to other pairs of atoms. In Heisenberg's theory of ferromagnetism, on the other hand, it must be assumed that J is positive in order to make the spins want to be parallel and so reinforce each other. Again the exchange effect plays an heroic role, as it supplies a mechanism for the lining up of the magnets, making the Weiss molecular field no longer a mystery. The coupling which Weiss had to assume between magnets was far too large to have a magnetic origin, but now the exchange effect (1) supplies the missing link between the magnets. The fact that ferromagnetic media are rather rare suggests that the case of positive J is a bit uncommon, and that the Heitler-London assumption $J<0$ is usually warranted.

AFTER this introductory and explanatory apotheosis of the exchange effect, we shall now turn to some current papers in the Physical Review on inter-atomic forces. Dr. Heitler[1] notes that in complicated compounds it is impossible to say that two particular electrons are accurately paired to zero resultant spin. The reason is that a free spin of one atom would like to be paired with the free spins of all other atoms, and has difficulty in choosing a particular partner. (By a free spin we mean, of course, one not already mated in a pair representing a saturated valence.) However, Heitler finds that it is possible to calculate quantum mechanically the fraction of the time that a given electron selects some other electron as a partner with which to be paired. As an example let us consider a molecule of the form $H_1A_1A_2H_2$, where A_1, A_2 are atoms having two, and H_1, H_2 atoms having one valence electron each. He finds that the pairing is as follows:

64 percent of time (A_1, A_2) (A_1, H_1) (A_2, H_2)
2 percent of time (A_1, A_2) (A_1, H_2) (A_2, H_1)
34 percent of time (A_1, A_2) (A_1, A_2) (H_1, H_2).

Here members of a pair have been inclosed in parentheses and labelled by the atoms to which they belong; the letters A_1 and A_2 of course appear twice on each line because atoms A_1 and A_2 are each represented by two valence electrons. If we use a chemical structural formula in which only bonds between adjacent atoms are printed, the molecule $H_1A_1A_2H_2$ is thus seen to be predominantly H_1–A_1–A_2–H_2, with practically no traces of H_1A_1–A_2H_2 but with considerable indications of $H_1A_1=A_2H_2$. More generally, it often turns out that an electron of a particular atom has a quite marked preference for an electron of some other particular atom, even though occa-

[1] W. Heitler, "Quantum Theory and Electron Pair Bond," Phys. Rev. **38,** 243 (1931).

sionally deserting in favor of other partners. When this is the case it is possible to localize the valence bonds and construct simple structural formulas (e.g. H_1–A_1–A_2–H_2 in the example cited). When, however, the electron has migratory affections and approximately equal preferences to be paired with the electrons of a variety of atoms, no simple diagrams are possible. In this connection it must be mentioned that the quantitative calculations of Heitler are only very rough because he treats all electrons as *s* electrons whereas actually *p* or even *d* electrons are usually involved in complicated compounds, so that one ought really to calculate the coupling between orbital as well as spin angular momentum vectors, thus introducing the concept of "orbital" as well as "spin" valence.

Equations of State of Gases

THE Eq. (1) based on exchange forces has an important application not only to chemical binding but also to the imperfections of a gas whose atoms do not combine chemically since the various spins have already found satisfactory partners in their own atoms. If one makes the usual assumption $J<0$, Eq. (1) gives a repulsive force between atoms which have exclusively closed shells. This is as it should be, as we all know that atoms of inert gases and the like are scarcely on speaking terms with each other. To show that (1) gives such a repulsion we have merely to note that in a closed shell we may regard half of the spins as pointing north and half south. Hence it is equally likely that the spin of an electron in a closed atom be parallel or antiparallel with respect to the spin of an electron in another closed atom. Thus the second term of (1) vanishes on summing over all possible pairings between the electrons of two closed atoms. Then there remains only the first term of (1), which is positive if $-J>0$. So the exchange energy of the two closed atoms is positive and such atoms tend to avoid being together. This survival of the first term of (1) furnishes the repulsive forces between atoms in kinetic theory, which have previously been a mystery. The exchange integral J in (1) is found to vary exponentially with the interatomic distance, and hence the repulsive forces die off quite rapidly when the separation is large (more rapidly than most of the other forces described in the next paragraph), and come suddenly into play if the atoms are squeezed too tightly together. This explains why the concept of a molecular diameter has been useful in kinetic theory even though an atom is a planetary system rather than a billiard ball.

One must not form the impression that the exchange forces derived from (1) are the only forces between atoms. There are, of course also the ordinary polarization attractive forces well known in kinetic theory since they can be pictured in classical mechanics. These forces vary as the inverse seventh power of the distance in the case of neutral atoms composed entirely of closed shells. Further, one has the ordinary Coulomb forces if atoms are ionized, and the Coulomb energy of interpenetration between neutral atoms if they are brought so close that their charge clouds intermingle.

By superposing the repulsive exchange forces, and the polarization and interpenetration attractive forces, great progress has recently been made in accounting for the equations of state of gases.[2] In particular, in the case of helium, where the computations are most tractable, the agreement of the calculated second virial coefficient with experiment is quite good over a considerable range of temperatures, although deviating at low temperatures.[2] Such things as the Joule-Thomson effect for helium, and the correction to a helium gas thermometer to yield the thermodynamic temperature can now really be calculated theoretically. Very recently, Margenau[3]

[2] T. C. Slater and J. G. Kirkwood, "The Van der Waals Forces in Gases," Phys. Rev. **37**, 682 (1931); J. G. Kirkwood and F. G. Keyes, "The Equation of State of Helium," Phys. Rev. **37**, 832 (1931); H. Margenau, "Note on the Calculation of Van der Waals Forces," Phys. Rev. **37**, 1425 (1931); also earlier work by Wang, Phys. Zeits. **28**, 663 (1927): Eisenschitz and London, Zeits. f. Physik **60**, 491 (1930); London, ibid. **63**, 245 (1930). H. Margenau, ibid. **64**, 584 (1930); H. R. Hassé, Proc. Cambr. Phil. Soc. **27**, 66 (1931).

[3] H. Margenau, "Surface Energy of Liquids," Phys. Rev. **38**, 365 (1931).

has shown that the superposition of these various forces which accounts for the imperfection of gases also accounts for the surface energy of the liquids, although the results are now more in a qualitative than quantitative stage. These various papers mark the start of a real atomic kinetic theory of gases and liquids in which forces between the atoms are calculated dynamically from the electron structure without undetermined constants. This is to be contrasted with the older work in which the forces between atoms were taken arbitrarily to vary as the nth power of the distance, etc. Less progress has been made in analysing the cohesive forces which hold the atoms of solids together, but they are commencing to show signs of being vulnerable to theoretical attack. Some previous work of Slater's suggests that possibly here the exchange forces may be rather less important than usual.

Classical Approximations in Quantum Mechanics

WITH its exclusion principle, exchange effect, etc. the non-relativistic quantum mechanics is now so perfect that, to quote Dirac, "The underlying physical laws necessary for the mathematical theory of a large part of physics and the whole of chemistry are completely known, and the difficulty is only that the exact application of these laws leads to equations much too complicated to be soluble." To have a quantitative theory of chemical reactions, collision phenomena, etc., what is needed is thus not improvements in principle, but methods of simplifying the calculations without impairing accuracy excessively. As classical theory is merely a special asymptotic case of quantum mechanics, it is thus important to examine whether classical analysis may not be a legitimate substitution for quantum mechanics in certain degrees of freedom, even though quantum refinements are needed for some of the other degrees of freedom in the same problem. It is interesting to note that two or three current papers in the Physical Review deal with the use of classical approximations.

One of these papers, by Slater,[4] deals with the legitimacy of treating the translational motions of atoms by means of classical theory when the imperfections of a gas are studied in the light of the various inter-atomic forces (exchange, polarization, and Coulomb). Suppose that first we anchor down two atoms so that their centers of gravity are at a constant distance r from each other, and calculate quantum mechanically the interactions of their electrons. Next we can move the anchors so that they are separated by a different distance r. By repeating this, we can obtain the quantum energy of interaction as a function of r, which we shall denote by $W(r)$. If, finally the anchors are removed, the atoms will be subject to a potential energy $W(r)$, and so will attract or repel each other, making the problem become kinetic rather than static as regards the motion of the centers of gravity (nuclei) of the atoms. Strictly speaking, this motion should be analysed quantum mechanically, but unfortunately all quantum collision problems are rather difficult. Therefore in various recent theoretical papers on the imperfections of gases, cited above, the forces between atoms are calculated quantum mechanically, but the translational motions of the atomic centers subject to these forces and the statistics of the relative prevalence of different distances of separation, such as are involved in calculation of virial coefficients, etc. are attacked by means of classical theory. Slater now examines whether this is really legitimate. He concludes that it is allowable, except possibly for very light atoms. The translational energy is of the order kT, and with atoms which do not unite chemically, the quanta of vibrational energy are so small that kT corresponds to quite a large number of vibrational quanta, and so the classical approximation is adequate. On the other hand, Slater concludes that use of the classical instead of quantum statistical treatment of the translational motions may possibly explain why the calculated second virial coefficient for helium is somewhat too low at low temperatures, though agreeing quite well with experiment at higher temperatures.

A somewhat similar point on the use of classical theory for translational degrees of freedom is made by Dr. Zener[5] in connection with the excitation of atoms at collision. He

[4] J. C. Slater, "The Quantum Theory of the Equation of State," Phys. Rev. **38**, 237 (1931).

[5] C. Zener, "Low Velocity Inelastic Collisions," Phys. Rev. **38**, 277 (1931).

concludes that even though the internal readjustment of the atoms (disturbances of electrons, etc.) must be treated quantum mechanically, it is sometimes a fair approximation to study the motions of the centers of gravity of the two colliding particles by means of classical theory. This yields information not deducible by ordinary quantum perturbation theory if the molecules are moving too slowly. Thus on Zener's view, two colliding molecules might be likened to two houses colliding in a tornado such that the motion of whole houses can be analysed classically, whereas the quantum mechanics is necessary in order to find out what is going on inside the house,—whether any inmates (electrons) are shaken out a window by the collision (ionized) etc.

Because of the exchange effect, it is usually necessary to calculate quantum mechanically the forces between atoms. However, in certain instances, some information on chemical bonds can be obtained by means of a classical calculation in which merely the Coulomb and polarization forces are invoked. Such instances are found mostly in compounds in which the constituents are ions rather than neutral atoms, as the Coulomb forces are much larger between ions than between neutral atoms. Slater[6] uses a purely classical calculation (except for knowledge of the ionization potentials of individual atoms) to show the structural formula for radicals such as NO_3^- is probably predominantly $N^{++}(O^-)_3$ rather than $N^{5+}(O^{--})_3$. He also gives quantum arguments based on orbital valence to supplement Zachariasen's classical polarization considerations to explain why the latter's x-ray data indicate that radicals with 24 electrons (e.g. NO_3^- form plane configurations, while those with 26 electrons (e.g. ClO_3^-) form pyramids.

[6] J. C. Slater, "Note on the Structure of the Groups XO," Phys. Rev. **38**, 325 (1931).

Photographic Models of the Charge Distribution in Atoms

PHOTOGRAPHIC models illustrating the charge distribution in atoms, which is proportional to the square of the magnitude of the Schroedinger wave function ψ seem to be much in style these days. A model is constructed in which the role of the electron is played by an object which appears as a luminous cloud on the photographic plate. The usual illustrations, such as those by Langer and Walker in Ruark and Urey's book, have been made exclusive of the spin corrections. Now H. E. White[7] is publishing some photographs of models based on the Dirac electron and so inclusive of the spin corrections. This modifies the angular, but not appreciably the radial distribution. In looking at these photographs one must bear in mind that they are merely time exposures of what an electron is doing in a stationary state, for by the Heisenberg uncertainty principle we abnegate all information on the instantaneous position of the electron, i.e. phase in its orbit, as soon as we assign it to a state of determinate energy. One should particularly guard against the impression that the electron is a fluid-like mass just because the time exposure reveals a luminous cloud. This would be to commit the same fallacy as to infer from the cloud-like appearance of a long time exposure of the motion of a firefly on a dark night that the firefly is a luminous fluid pervading a large space (its actual region of motion) rather than a discrete bug. From the time exposure one cannot locate the firefly instantaneously, but one can infer its favorite haunts, as these are the regions of greatest luminosity on the photographic plate. Mr. White's ingeneous mechanical devices are tantamount to photographing an electron as one would a firefly, and the bright regions in his pictures are the favorite haunts of the electron. The brightness is, of course, proportional to the statistical charge density $-e|\psi|^2$. The photographs, to be sure, reveal nothing really new, as ψ is already known by solving the wave equation. Otherwise the model could not be constructed. However, it seems to be human nature to like photographic illustrations, even though the photographs are necessarily of a model rather than of a real atom.

Instead of a time exposure of the quantum model, one could in principle equally well make a time exposure of a model representing the motion of an electron in a classical orbit. There would be complete blackness in the regions for which the potential energy V exceeds the total energy W, as this would demand neg-

[7] H. E. White, "Pictorial Representations of the Dirac Electron Cloud for Hydrogen-Like Atoms," Phys. Rev. **38**, 513 (1931).

ative kinetic energy. In the region of positive kinetic energy, the classical brightness would be inversely proportional to the velocity. This is because the electron lingers longest where it moves slowest. The time dt taken to traverse a distance dx is $dt=(dt/dx)dx$, and hence inversely proportional to the velocity dx/dt. (We assume for simplicity only one degree of freedom.)

Transmission Coefficients in Thermionics

THE comparison of the classical and quantum statistical charge densities is interesting. The photograph of a classical model would reveal a perfectly sharp shadow or cutoff at the positions where the kinetic energy changes sign. On the other hand the luminosity in the quantum picture tails off gradually, and there is an appreciable, though small, brightness even in the regions of negative kinetic energy. This means that in quantum mechanics a particle can surmount barriers of potential energy which it could never leap in classical theory. The important and well-known applications of these ideas to radioactive and thermionic emission we shall not attempt to discuss here. Usually it has been necessary to assume barriers of constant slope (rectangles, polygons, etc.) in the interest of analytical simplicity. Frank and Young,[8] however, show that the calculation of transmission coefficients in thermionics for hills of arbitrary shape can be made with an approximation often sufficient by utilizing the fact that an asymptotic solution of the Schrodinger wave equation, valid especially for large quantum numbers, is $\psi=(W-V)^{-1/4}e^{2\pi iS/h}$, where S is a function (the so-called action function) which is real inside the classical libration limits, and a pure imaginary elsewhere. This form of asymptotic solution is well known in the literature, and sometimes called the Wentzel-Kramers-Brillouin approximation. It was utilized by Gurney, Condon, and Gamow to calculate transmission coefficients in radioactive decay, and now it is employed by Frank and Young to yield the Fowler-Nordheim formulas for thermionic emission. Various writers have noted that this W.K.B. solution, though approximate, possesses sufficient refinement to reveal many interesting properties of the true wave function. As $\frac{1}{2}mv^2=(W-V)$, and as $|e^{2\pi iS/h}|=1$ if S is real, the W.K.B. expression for $|\psi|^2$ is proportional to $1/v$, yielding exactly the same probability as in classical theory. (A more complete investigation, however, shows that solutions with $+i$ and $-i$ should be superposed, leading to a factor cos $(2\pi S/h+\epsilon)$ instead of $e^{2\pi iS/h}$. This gives the wave phenomenon, and the envelope of the waves follows asymptotically the classical distribution curve. In other words, the quantum time exposure displays wave-like oscillations in intensity not found in classical theory). Outside the classical libration limits, one has instead $e^{2\pi iS/h}=e^{-2\pi|S|/h}$, and the presence of the exponential factor assures that there is usually not a large probability of an electron going where it is not allowed classically. This corresponds to the fact that in physical optics the wave intensity is usually not large beyond the geometric shadow, which is the dead-line in geometrical optics. The fact that the exponential factor does not vanish completely shows us that the electron can wander beyond the classical dead-line into the forbidden region of negative kinetic energy, and so we have such paradoxes as one volt electrons leaping 1.1 volt hills, etc.

[8] N. H. Frank and L. A. Young, "Transmission of Electrons through Potential Barriers," Phys. Rev. **38**, 80 (1931).